



REFLECTIONS

BELIECTIONS

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REFLECTIONS

BE REFLECTIONS?

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From The Editor's Desk

As the new REFLECTIONS editor, I want to begin this section by sending a thank you to Cheryl Hughes. Cheryl retired as REFLECTIONS editor this summer after seven years at the helm. Thanks Cheryl for all your time and dedication to REFLECTIONS!

As we are all aware 'GPS' is very much on everyone's mind these days. In upcoming issues, both I and the review board would like to publish articles that help with GPS implementation. Articles that focus on instructional strategies to teach GPS, differing assessment techniques that can be used to enhance student learning, student GPS task solutions and implementation issues, in general, are of interest. We want to hear from you!

In this issue, check out the updated conference information. This year the conference begins on Wednesday night and ends Friday. Please take the time to provide GCTM feedback regarding this change.

It is with great sadness we note the passing of Jeff Weeks this summer. Jeff's wisdom and enthusiasm will be greatly missed.

If you have any suggestions or comments regarding REFLECTIONS please email me at the new REFLECTIONS email address, reflections@georgiasouthern.edu. I look forward to hearing from the membership!

Gregory Chamblee
Editor

Author Guidelines

Manuscript Format: Manuscripts are reviewed by members of the editorial review board in a blind review. For this reason, each manuscript should include a cover sheet containing title of manuscript, author's name, position and email address. Identifying information should not appear elsewhere in the manuscript in order to ensure an impartial review.

Manuscripts should be double-spaced, with 1-inch margins on all sides, typed in 12-point font and follow the APA 5th Edition style guide. Manuscripts should be submitted in MS Word. If you have a picture or graphic in the text, please include the original picture(s) in a separate file.

Manuscript Submission: Manuscripts should be submitted to reflections@georgiasouthern.edu. Receipt of manuscripts will be acknowledged. Manuscripts are accepted for consideration with the understanding that they have not been published previously and are not being considered simultaneously for publication elsewhere. Additional inquiries should be sent to Gregory Chamblee, Editor, Georgia Southern University, Department of Teaching and Learning, PO Box 8134, Statesboro, GA 30460-8134; Phone: 912.681.5701; Fax: 912.681.0026; reflections@georgiasouthern.edu.

Manuscript Publication: When a manuscript is accepted for publication, the editor/editorial review board may make suggestions or revisions in consultation with the principal author. However, because of publication deadlines the editor reserves the right to make minor revisions without seeking prior approval from the author. Release statements for all copyrighted materials must be received prior to publication. Upon publication, two complimentary copies of the issue are sent to the principal author.



GCTM NEWS



Let's Grow Together

by Barbara Ferguson
GCTM President
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This issue of Reflections comes with the beginning of a new school year. Many of you attended NCTM Annual Conference in Atlanta this past March. Thanks to all the wonderful volunteers, mathematics teachers from all over the world had a great time—full of enjoyable and educational experiences! Now this fall I hope you will all attend the 48th Annual Georgia Mathematics Conference at Rock Eagle.

Conferences are great opportunities to learn, interact with other mathematics teachers, be wooed by the latest technology and classroom materials, and receive activities to use with your students right away. But more importantly, teachers have the opportunity to ponder their own teaching. Sessions emphasize the mathematics that underlies the student activities. The activities don't do the teaching, but they give a context to examine and discuss and help students make the mathematics explicit. The analysis and discussion are where the real work happens.

As teachers give of themselves to instruct their students, it is important to reverse that role and receive instruction. Teachers need

the opportunity to be a learner and bring renewed energy back to their students. New ideas and new ways to present material are vital. The post session conversations help to focus and clarify presentations. The time spent can be most rewarding, not only educationally, but also socially as we share good times with old and new friends! Remember that "harvest time" truly lies in the beginning of the school year as we lay the foundation for a successful year. While you spend time helping your students be better learners, please give back to yourself. If the Georgia Mathematics Conference at Rock Eagle is not an option, fall is a great time to begin some type of on-going professional development for you and your colleagues. With the Georgia Performance Standards still "rolling out," teachers need to support one another. I would like to encourage you to form a support group in your school. Decide what you want to accomplish this school year and what will work best in your school. However, regardless of what you choose, it is most important that YOU take the initiative to get the group going and keep it going throughout the year.

Many mathematics educators, and their families, give hundreds of volunteer hours in behind the scene efforts that make the GMC at Rock Eagle a meaningful experience. The committee chairs and their members contribute countless hours of time and energy to bring over 1,800 mathematics educators together. Another component that contributes to the successful conference is the rich content provided by all of the speakers who spend time to submit proposals and then more time in preparing their presentations. These presentations provide opportunities for reflection and stimulation as well as hands on opportunities for learning. However, no conference can be successful without having mathematics educators in attendance. I recognize how much work it takes to get ready for a substitute, and then how much more time it takes upon returning from the conference. But again I emphasize the importance of your professional growth. Join us at Rock Eagle this October — LET'S GROW TOGETHER!!

Did You Know?

by Becky King
GCTM Executive Director
bwking@comcast.net

The first Georgia Mathematics Conference was a one-day meeting on Saturday, Feb. 15, 1958. The program for the day consisted of an opening general session followed by two sessions each for elementary and secondary teachers. During the night of Feb. 14, Georgia had a near record snowfall. One hundred and forty people had made reservations for lunch, but no money had been collected and the GCTM treasury had only \$18.75! Despite the bad weather, 110 teachers attended that day. Since that successful beginning, money has been collected with advance registrations. What a long and wonderful history we have, and how thankful we are to those who have paved the way before us.

Earning a PLU Credit Requires Three Steps:

Step 1 is done **before** the conference. The PLU Prior Approval Form for 2007 is now on the GCTM Web site. Visit www.gctm.org to download the form. Fill it out and get the required signatures **before** you attend the conference. (Do not include this form with your preregistration.)

Step 2 is done **at** the conference. Bring the prior approval form to the PLU desk in the registration area at the conference and receive a form to be filled out as you attend sessions.

Step 3 is done **after** the conference. This is an on-the-job assessment completed in your school system. The purpose of this element is to put to use what you have learned. You will receive the proper form for this at the conference. If you have questions about earning PLU credit at the conference, contact Becky King via email at bwking@comcast.net.

2006 GCTM Teacher Promise Award Recipient



The 2006 GCTM Teacher Promise Award was presented to Amanda Russell of Fayetteville Intermediate School in Fayetteville. This award is given to a teacher who is in the first three years of a career and who has shown outstanding promise in teaching mathematics. The award was presented to Amanda at a gathering of her colleagues in Fayetteville and was given by Lynn Ridgeway of the GCTM Executive Board.



GCTM NEWS



Jeffrey Thomas Weeks

In Memoriam July 21, 1954 - July 18, 2007

by Bill Marsh
bnmarsh335@comcast.net

The Georgia Council of Teachers of Mathematics lost a stalwart member a few weeks ago. Jeffrey Thomas Weeks known to most of us as Jeff died following heart surgery to replace a valve.

Jeff was a life member of GCTM and a life long supporter of mathematics education at all levels. He became the NE Regional Representative in 1990 and then in 1995 was elected VP of Constitution and Policies. Those of us who worked with him knew him as the VP of “stuff”. The 90s was the time the Constitution was totally revised, we became an incorporated organization and began to organize our charitable foundation. During those years we had to revise the Policies to reflect our new Constitution and Jeff was one of two or three that spent hours and hours revising the policies and then insisting we follow them. Often a meeting stopped and changed direction as we were told the topic violated a particular policy.

There is no subject that Jeff didn’t teach at the middle or secondary levels. He just loved to teach! In recent years he taught at North Clayton High School and more recently in several high schools and middle schools in Cobb County.

The July 23, 2007 funeral was held at St. Luke’s Episcopal Church in downtown Atlanta where he was an active member who frequently sang in the choir. The number of former students present during the service was amazing and a testament to his love for teaching and caring for students.

Sometimes life just ain’t fair and we truly lose a “good one.”

A New Year -- New Members

by Susan Craig
GCTM Membership Director
SECDDC@aol.com

Days of summer slip away like water through our fingers. It seems we are finalizing grades and storing classroom materials one day and suddenly we are back in the swing of things with memories of vacation fun and summer tomatoes and peaches. I hope your memories are wonderful ones and your new year the best yet! If you are beginning your career, make a great memory every day— for you and your students!

Each year at this time I have encouraged GCTM members to invite a colleague to join us in our professional organization. I have often been surprised when I have extended such an offer to new teachers and colleagues. Often they have not heard of GCTM or realized the benefits with such a small fee attached. So it is easy to be an ambassador. GCTM is growing--there are 75 new members on the report than in the spring! Share GCTM with all your colleagues.

If you would like materials to share with other teachers, GCTM can provide brochures and membership forms. Just contact me at SECDDC@aol.com. Remember you can find your membership number and renewal date on the mailing label on all publications.

GCTM Membership Report											
	<u>NW</u>	<u>NE</u>	<u>MW</u>	<u>ME</u>	<u>CW</u>	<u>CE</u>	<u>SW</u>	<u>SE</u>	<u>Out of State</u>	<u>Lost Members</u>	<u>Totals</u>
Annual	181	178	162	227	161	237	190	245	8	8	1597
Life	31	43	80	41	68	61	40	62	12	39	477
Student	121	85	97	30	18	310	8	36	0	0	708
Total	336	306	339	298	247	608	238	343	20	47	2782

2007 GCTM Election Results



President-Elect
Lynn Stallings



Vice-President for
Regional Services
Peggy Pool



Vice-President of
Constitution and Policies
Blanche Presley



Secretary
Patti Barrett

GCTM NEWS



GCTM Mini-Grant Information

by Melissa Walker
V.P. of Regional Services

Greetings Fellow Georgia Mathematicians!!!

Did you know that GCTM provides funds to support classroom teachers in the areas of **improving classroom practices** and **increasing teachers' mathematical knowledge**? The GCTM Mini-Grant Program is being implemented to provide funding for creative teaching projects.

Applications will be reviewed in September. Winners will be notified by early October and recognized at the Georgia Math Conference at Rock Eagle at the GCTM business meeting on Thursday, October 18th at 7:00 p.m. Proposals will be judged anonymously and grants will be awarded in any amount up to up to \$300.00.

The projects will be implemented from October through June. Each winner must submit an evaluation of his or her project by April 15th and must be willing to either write an article for *Reflections*, the GCTM publication, or participate on a panel with other Mini-Grant winners at the following Georgia Math Conference.

Applications need to be a lesson plan with activities for students, not just a request for materials. The criteria upon which this application will be evaluated are:

- Creativity, innovation
- Potential impact upon student achievement
- Potential for replication by and dissemination to other teachers
- Advancement of NCTM's *Principles and Standards for School Mathematics*
- Unavailability of funding from local sources
- Project evaluation plan

To apply for a Mini-Grant, you must return three copies of the application by **Friday, September 21st**. Send application (http://www.gctm.org/mini-grant_application.htm) to:

Missy Walker
Vice President of Regional Services
5844 Mays Ridge Court
Rex, GA 30273
Or by e-mail to: mwalker@gctm.org



GCTM Regional Reports



CENTRAL-EAST REGION



Amber Donnell

The Central East Region is planning a mini conference for a Saturday in February. Please email Amber Donnell at amberdonnell@lcboe.net with suggestions for speakers, topics, or if you are willing to volunteer to help with the conference. I hope to see you at Rock Eagle!

CENTRAL-WEST REGION

Teachers in the Central West Region have been busy taking advantage of professional development opportunities during the past few months. Many teachers from the region attended the NCTM National Meeting and Exposition in Atlanta in March. Several teachers from the region presented at the conference and a number of others served as volunteers. Almost 70 teachers from Muscogee, Harris, Troup, Talbot, and Chattahoochee counties attended the Columbus Regional Mathematics Collaborative's summer Teacher Quality workshop, June 11-15, at Columbus State University. Members of the Chattahoochee Council of Teachers of Mathematics are in the process of recruiting new officers and getting CCTM off to a fresh start for this school year. We attended our first GCTM Executive Meeting in Macon on June 16. We are slowly becoming familiar with our role as region representatives. Mathematics Educator, Dr. Doug Brumbaugh, will be the keynote speaker at the Columbus Regional Mathematics Collaborative's Fall MathFest on Thursday evening, October 4th and again at their Fall Workshop on Friday, October 5th. Dr. Brumbaugh is the author of the Scratch Your Brain Where it Itches problem solving books as well as numerous other mathematics publications. The events will be held on the campus of Columbus State University. For registration information email, crm@colstate.edu. Please contact us if we can do anything to help promote GCTM and mathematics education at your schools.



Kenneth Jones



Peter Anderson

GCTM NEWS



NORTH-EAST REGION

In light of our new state curriculum for mathematics, this is a wonderful time to be a part of GCTM. We encourage all mathematics educators in the NE GA Region to join us!

Recent Events:

- There have been two important GPS workshops for high school teachers that have been available this summer in our region:
 - o *Training for GPS – Mathematics I* was offered by the DOE & RESA on May 29th-31st, 2007
 - o *Teaching High School Mathematics in a Standards-Based, Integrated Classroom* was sponsored by NEGA PRISM, NEGA RESA, and Glencoe on July 23rd-24th, 2007 and July 26th-27th, 2007.
- PRISM learning communities have submitted 4 proposals to speak at the GMC at Rock Eagle in October, 2007.

Upcoming events: Here are some of the professional learning opportunities for mathematics teachers in the NE GA region:

- Two days of GPS training to follow-up on the work of the *Training for GPS-Mathematics I Workshop* will be September 25, 2007, and February 26, 2008
- *Teaching Conceptual Mathematics to the High School Learner* is a professional learning community for high school teachers which will begin Sept. 10th, 2007.
- *Connected Mathematics Learning Community* is offered for middle school teachers.
- The *K-2 Mathematics Initiative* and the *3-5 Mathematics Initiative* are planned for elementary teachers.
- *Teaching Conceptual Mathematics to the Middle School Learner* (year 2) is planned for Madison County middle school teachers and their collaborative teachers and is sponsored by Northeast Georgia GLRS.

For more information please contact Sandy Blount at sandyb@uga.edu or Kaycie Maddox at Kaycie.Maddox@negaresa.org.

NORTH-WEST REGION

On June 1, 2007, Mathematics Department of Kennesaw State University invited 4 publishers to come and present their mathematics curricula to a group of high school teachers who are piloting some of the High School Math I materials. These 4 projects were created through funding from the National Science Foundation and are standards-based curricula that require students to investigate mathematics, make and test conjectures, reason, make mathematical connections, problem-solve, and represent mathematics in a variety of ways. These publishers addressed the extent to which their books are aligned with the Georgia Performance Standards (GPS). About twenty high school teachers from around the Kennesaw area attended the workshop. They had the first opportunity to learn some really different and exciting products for input on new textbook adoption and GPS application.

The workshop lasted from 9:00 am to 3:00 p.m. at Clendenin building. The four curriculums presented are from the following publishers/textbook: It's About Time (Math Connections), Key Curriculum Press (Interactive Math Project), Glencoe & McGraw Hill (Core Plus), SIMMS Interactive Mathematics.



Jeni Halimun

GA ARML Among Best in Nation

by Debbie Poss
V.P. for Competition
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The Georgia All-State Math Team finished 5th in the nation and the Georgia B team finished 30th out of approximately 125 teams at the 32nd annual ARML (American Regions Math League) competition on June 2. This marks the 17th straight year that Georgia has finished among the top 10% at this “World Series” of mathematics tournaments for high

school students. Held simultaneously at Penn State University, the University of Iowa, and the University of Nevada at Las Vegas, this competition attracts 2000 students from across the United States, as well as international teams from Canada, Taiwan, Columbia, Turkey, and Kazakhstan. For the first time ever, Georgia students placed among the top ten individuals, with Howard Tong of Lakeside (Atlanta) High School placing second and Linda Liu of Parkview High School finishing sixth. One of the Georgia coaches, Steve Sigur of the Paideia School, was named outstanding coach of the year. Steve has been a coach of the Georgia ARML Team since 1992.

The members of the 2007 Georgia ARML team were selected based on their outstanding performance at the GCTM State Math Tournament, as well as the AMC12 and AIME competitions held in February and March. Trophy winning performances at other math tournaments held in Georgia throughout the year also was a consideration. Members include:

Scott Brothers — Walton High School
Sam Brotherton — Rockdale Magnet School
Harrison Brown — Centennial High School
Wesley Brown — Greater Atlanta Christian
Billy Dorminy — Sola Deo Gloria Home School
Mark Doss — Columbus High School
William Drobny — Parkview High School
Miles Edwards — Lassiter High School
Eddy Ferreira — Roswell High School
Robert Grosse — Chamblee Charter School
Nathan Hipsman — Lassiter High School
Benjamin Hu — Northview High School
Peter Huang — Lakeside (Evans) High School
Tony Huang - Northview High School
Santhosh Karnik — Wheeler High School
Nayoon Kim — Rockdale Magnet School
Michael Lai — Athens Academy
Ruby Lai — Athens Academy

Amy Lanchester — Rockdale Magnet School
Sean Lee — Centennial High School
Tomas Leon — Pope High School
Linda Liu — Parkview High School
Eric Morphis — Woodward Academy
Phillip Mote — Lassiter High School
Karthik Narayan — Northview High School
Jordan Ou — Chamblee Charter School
Sagar Patel — Brookstone Academy
Steven Rouk — Rockdale Magnet School
Kevin Sun — Centennial High School
Howard Tong — Lakeside (Atlanta) High School
Carol Wang — Chamblee Charter School
Brayden Ware — Mount de Sales Academy
Maja Wichowska — Alpharetta High School
Casey Woodrum — Southeast Bulloch High
Claus Zheng — Dickerson Middle School



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Coaches include Tom Fulton (Walton High School), Chuck Garner (Rockdale Magnet School), Ben Hedrick (Alpharetta High School), Charles Koppleman (Kennesaw State University), Adam Marcus (Georgia Tech), Debbie Poss (Lassiter High School), Steve Sigur (Paideia School), and Don Slater (Lassiter High School).

The American Regions Mathematics League's annual competition brings together the nation's finest students, where they meet, compete against, and socialize with one another, forming friendships and sharpening their mathematical skills. The contest is written for high school students, although some exceptional middle school students attend each year. Three of the four parts of this meet involve teamwork such as the Power Question, which is a one-hour challenge where each team writes solutions and proofs to a complex, multipart problem. (This year's topic dealt with continued fraction expansions, using a process analogous to the Euclidean algorithm.) Other parts of the competition include a Team Round where the team is given 20 minutes to answer ten questions, and a Relay Round where students pass their answers on to teammates who need it to solve their problems. The competition also features an Individual Round where each student independently solves eight difficult questions.

Samples of questions from this year's competition include: (*answers below*)

Team #2: Determine the number of three-digit positive integers such that if the integer is divided by the sum of its digits, the result is 19.

Team #6: In a right triangle with integer sides the radius of the inscribed circle is 12. Compute the longest possible hypotenuse.

Individual #5: Let $[x]$ denote the greatest integer function and let x be in radian measure. Compute all x in the interval $0 \leq x \leq 2\pi$ such that $f(x) = \sin [2x]$ takes on its largest value.

National sponsorship for ARML is primarily provided by the D. E. Shaw group, a specialized investment and technology development firm. For more information about the competition go to www.ARML.com

*Answers to sample problems above:

Team #2: 11

Team #6: 313

Individual #5: $4 \leq x < 4.5$

Call for Reviewers

The journal is in need of reviewers. If you have an interest in reviewing please send your name to reflections@georgiasouthern.edu.

GCTM Math Tournament News

by Debbie Poss
V.P. for Competition
dposs@gctm.org

The GCTM State Math Tournament was held at Mercer University on April 28, 2007. The 32 teams of four mathletes joined about a dozen individuals, all invited based on their performance on previous Georgia math tournaments held during the 2006-07 school year. After a challenging competition consisting of a written test, individual ciphering and pair ciphering, awards were given to the top 10 individuals and to the top 5 teams. Trophies went to:

Top Individuals:

1. Phillip Mote – Lassiter
2. Harrison Brown – Centennial
3. William Drobney – Parkview
4. Sam Brotherton – Rockdale Magnet
5. Nathan Hipsman – Lassiter
6. Amy Lanchester – Rockdale Magnet
7. Brayden Ware – Mount de Sales
8. Nayoun Kim – Rockdale Magnet
9. Santosh Karnik – Wheeler Magnet
10. Billy Dorminy – Soli Deo Gloria

Top Teams:

1. Lassiter High School
2. Rockdale Magnet School
3. Centennial High School
4. Chamblee Magnet School
5. Northview High School

Sample questions from the 2007 GCTM State Tournament are:

15. How many five-digit positive integers have the property that the product of their digits is 1000?

A) 20 B) 40 C) 60 D) 80 E) 100

24. Two 6-sided dice have their faces painted either yellow or purple. The first die has 5 yellow faces and 1 purple face. When the dice are rolled, the probability that the top 2 faces show the same color is Ω . How many purple faces are there on the second die?

A) 1 B) 2 C) 3 D) 4 E) 5

31. What is the largest possible angle measure, in degrees, of an angle formed by joining the midpoints of three edges of a cube?

A) 60 B) 90 C) 120 D) 135 E) 150

Ciphering #3. In a sequence of numbers the sum of the first n terms is given by n^2 . What is the 2007th term of this sequence? (4013)



GCTM NEWS

GCTM Math Tournament News

The 2007 Middle School Tournament was held on April 21, 2007 at Thomson Middle School near Warner Robbins. Eleven schools brought up to 8 students to compete in a written test, individual ciphering questions, pair ciphering rounds and a power question in which all team members worked together. After pizza and the frightening lightning round (which doesn't count, but does provide a candy prize to all participants), the top 10 students and top 5 teams were announced. Winners were:

Top Individuals:

1. Claus Zheng – Dickerson
2. Alex Li – Dickerson
3. Evan Place – First Presbyterian
4. Gil Goldshlager – Dickerson
5. Jake Montgomery – Greater Atlanta Christian
6. Joe Sheehan – Trinity
7. Minh Le – Dickerson
8. Zachary Shealy – First Presbyterian
9. Ben Scurry - First Presbyterian
10. Sarika Reddy – Fayette County

Top Teams:

1. Dickerson Middle School
2. First Presbyterian Day Middle School
3. Greater Atlanta Christian Middle School
4. Fayette County Middle School
5. Dodgen Middle School

Call for Manuscripts

Deadline for Winter 2008 Issue: November 15, 2007

Topics:

GPS implementation manuscripts are needed. For example, instructional strategies to each GPS, GPS implementation issues, working with special populations in a GPS environment and sample student task solutions are some of the ideas of interest.

Teaching Tips Ideas:

Share with your fellow teachers a pearl of instruction or assessment wisdom you have used in your classroom. Topics include how to design and implement effective warm-ups, strategies for implementing journal writing, etc. Manuscripts published in this section are typically one page in length.

Dr. Rock's Math Mystery Solutions:

Submit student solutions to Dr. Rock's Math Mystery. A sample of solutions submitted will be published in the next issue.

Making Mathematics FUN

Dr. Rock's Math Mystery

by David Rock
Columbus State University
rock_david@colstate.edu

Elementary Brain Teaser

From Last Issue

Handy Man solution: House numbers.

Explanation: Each number costs one dollar. In other words, if your address is 226 Lamar, and you have to put numbers on your mailbox, it would cost you three dollars for 226 since each number decal costs one dollar. If your address is 1245 it would cost you 4 dollars.

New One!

The Frog in the Pond.

There is a circular pond that is 300 feet in diameter. Dead in the center of the pond is a frog on a lily pad. If the average frog can leap two feet and there are plenty of appropriate spaced lily pads in the pond on which to jump, exactly how many leaps will it take the frog to get completely out of the pond? Give it a try! By the way, the answer is not 150 leaps and it is not 75 leaps either! Pass this along to your family and friends. Someone will unlock the key for you!

Challenge Round

From Last Issue

Checkerboard Solution:

204 squares.

Explanation: You have one 8x8 square, four 7x7 squares, nine 6x6 squares, sixteen 5x5 squares, twenty-five 4x4 squares, thirty-six 3x3 squares, forth-nine 2x2 squares, and sixty-four 1x1 squares.
 $1+4+9+16+25+36+49+64=204$.

New One!

Last Digit

What would the units digit be for 3 raised to the 9999th power?

The 48th Annual Georgia Mathematics Conference

*by Don Brown
Conference Board Chair*

The 48th Annual Georgia Mathematics Conference will be held at the Rock Eagle 4-H Center, its usual location, outside Eatonton, GA. This year's theme is CSI Mathematics: Celebrating Successful Instruction. The dates of the conference are October 17 – 19, 2007. This year's conference offers a slightly different format than years past. The conference will start on Wednesday, October 17, with an annual dinner compliments of Glencoe McGraw-Hill followed by a special opening session to kickoff the conference. The Wednesday night Opening Session is a new aspect of the conference and it will feature State Superintendent, Kathy Cox. Following the Wednesday night Opening Session will be a social activity and reception for you to visit with friends and colleagues. Thursday, October 18th, will provide opportunities for you to attend sessions, visit the exhibitors, participate in the annual Fun Run, and congratulate awards winners after the evening session. There will be more sessions on Friday, October 19th. This year's conference will conclude on Friday with a closing session starting at 3:30 pm.

The following is a list of keynote and featured speakers with a brief biography.

Keynote Speakers



Kathy Cox is the State Superintendent of schools. She will kickoff this year's conference with an opening session Wednesday night. She has been a classroom teacher for 15 years and served two terms in the Georgia legislature as a state representative. Ms. Cox's fundamental belief is "that all children can learn and deserve a high quality education."

Matt Larson is the K-12 math curriculum specialist for the Lincoln Public Schools in Lincoln, Nebraska. He is a frequent national speaker on issues related to improving mathematics achievement. He has coauthored two mathematics textbook series. Matt serves on NCTM's Research committee and NCTM's Linking Research and Practice committee.



NO
PHOTO
AVAILABLE

Michael Hibbard served as Assistant Superintendent for Curriculum and Instruction at Ridgefield public school system in Connecticut. Currently, he is working with the North Salem school district in New York.

Featured Speakers

Sybilla Beckmann has received degrees from Brown University and The University of Pennsylvania. Her experience includes teaching and research at Yale University and The University of Georgia where she is currently a member of the faculty. Sybilla has authored textbooks for prospective teachers of mathematics.

Jean Linner is a winner of the Excellence in the Teaching of High School Mathematics Award. She has degrees from The University of Florida, Georgia State University and Kennesaw State University, where she is currently pursuing a Master's degree with a concentration in mathematics. Jean Linner teaches mathematics at Lassiter High School in Marietta, Ga.

Brent Loeffle is a part of the Georgia Sheriff's Association.

Cherie Long is a winner of the Presidential Award for Excellence in Mathematics and Science Teaching. She has a Master's degree from The University of West Georgia in Middle Grades Education with a concentration in mathematics and science. Cherie's teaching goal is to "teach students to see that mathematics is all around them in the everyday world."

Irina Lyublinskaya is an associate professor at College of Staten Island in New York and co-director of The Discovery Institute. She has been teaching mathematics and physics for more than 20 years at both the high school and college levels. Irina presents regularly at regional and national conferences on mathematics and science. She has received many grants for curriculum development, teacher training and research. Irina has authored several books and articles.

Paula Snowdy is a member of the Marilyn Burns Education Associates which is dedicated to improving the teaching of mathematics at the K – 8 level through a series of professional development services and materials.

High School Math Tournaments Available to GA Students

by Chuck Garner
Rockdale Magnet School
cgarner@rockdale.k12.ga.us

A Path to Mathematical Excellence: Opportunities for Mathematically-Gifted High School Students

The state of Georgia is fortunate to have an active mathematical community that encourages the mathematically gifted and talented student. There are fifteen math tournaments for high school students held annually at various locations around the state and another seven annually held in neighboring states. This figure does not include the two state tournaments for Junior Varsity and Varsity teams. A team's placement at a tournament helps determine which teams get invited to the Varsity State Mathematics Tournament, sponsored by the GCTM.

The American Mathematics Competition is a worldwide contest that is offered in three levels: the AMC8 (for 8th graders and below), the AMC10 (for 10th graders and below), and the AMC12 (for 12th graders and below). The purpose of the AMC is "to identify and encourage, through friendly competition, students with an interest in and talent for mathematical problem solving." High-scoring students on the AMC are invited to take the American Invitational Mathematics Exam (the AIME). For Georgia students, the combination of results on these two contests may result in a personal invitation to the Varsity State Math Tournament.

The USA Mathematics Olympiad (USAMO) is the premier individual student event in the US. Based on combined AMC and AIME scores, approximately 220 students in the US are invited to take this contest annually. From this pool, the 28 highest-scoring students are chosen to attend the Mathematics Olympiad Summer Program (MOSP). The purpose of MOSP is to select the six-student team that will represent the United States at the annual International Mathematics Olympiad (IMO). The USA Mathematics Talent Search (USAMTS) is sponsored in part by the NSA, and is another route toward taking the USAMO.

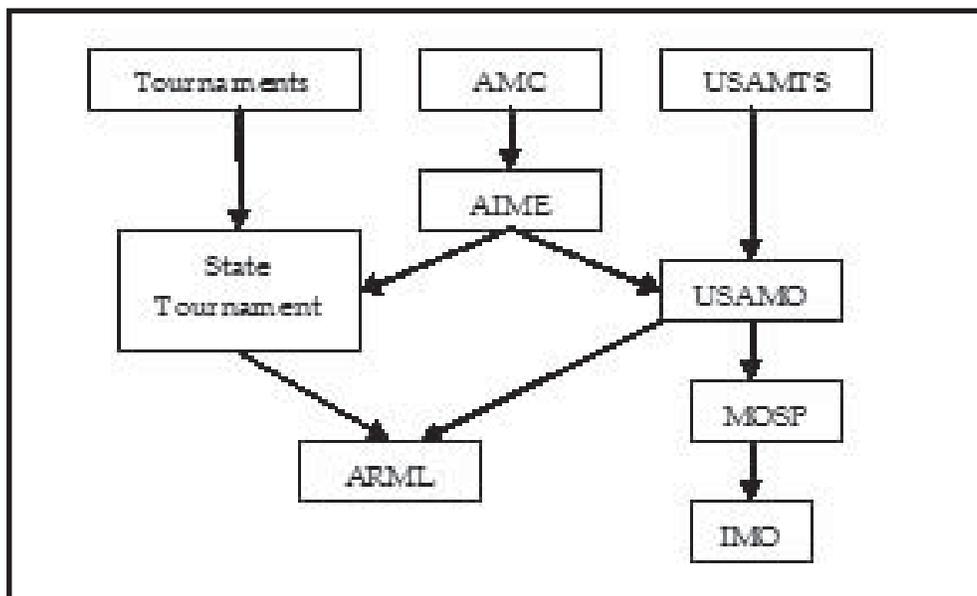
Quick Facts

AMC	75 minutes, 25-choice questions (Feb)
AIME	3 hours, 15 questions (Mar)
USAMTS	5 proofs once a month for 4 months (Oct-Mar)
USAMO	Two 4.5 hour sessions in 2 days, 6 proofs (Apr)
State Tournament	90 minutes, 50 multiple-choice questions, 10 ciphering questions, 8 team questions (Apr)

The AIME, USAMTS, USAMO, and State Tournament are FREE!

The IMO is the pinnacle individual achievement for the mathematically talented; however, the pinnacle team achievement is participating in the American Regions Mathematics League (ARML). Any well-defined geographic region may form a 15-member team (city, county, state, etc). In Georgia, we select the best 32 students in the state (as determined by the AMC/AIME/USAMO and the Varsity State Tournament) to form two 15-student Georgia teams, plus two alternates. Unlike the USAMO or IMO, students are required to work together at ARML, and this makes ARML in some ways more challenging.

Pathways to ARML and the IMO in Georgia



To aid in preparing the students for these challenging contests, many books and websites have been prepared. The most useful is probably *The Art of Problem Solving* series of books and websites. Some locales offer “Math Circles”—a monthly meeting of mathematically talented students run by a college professor. For instance, Dr. Harold Reiter at UNC Charlotte runs the “Charlotte Math Circle” for high school students in and around Charlotte.

Resources

AMC/AIME/USAMO/MOSP/IMO: <http://www.unl.edu/amc/>

USAMTS: <http://www.usamts.org>

Georgia Tournaments: <http://www.gctm.org>

ARML: <http://www.arml.com>

For Teachers and Students: <http://www.artofproblemsolving.com>

Georgia ARML: http://paideiaschool.org/TeacherPages/Steve_Sigur/GAARML/GEORGIAARML/Welcome.html

Judy's Book Shelf

by Judy Chambers
Retired

judychambers100@hotmail.com

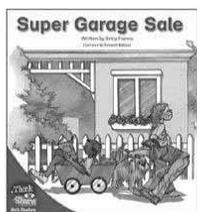


A Second Is A Hiccup, A Child's Book of Time by Hazel Hutchins ©2004. This is one of the most delightful books I've seen in a long time. If you are teaching PreK-1st grades and you are looking for something that will help children develop a sense of time, get this book. It explains the concept of time in terms that any child can understand. "A second is a hiccup, the time it takes to kiss your mom or to turn around." How long is a month, if you fall down and scrape your shin, in a month, there's new skin. This book is a must have!



Another new book out worth checking is **It's Probably Penny** by Loreen Leedy, ©2007. Ms. Leedy has another wonderful book, this time taking on probability. Lisa's teacher tells the class that they are going to study probability by making predictions and following

up with the results. Lisa decides to use Penny, her Boston Terrier for this lesson. Will Penny want to go for a walk, etc.? Between the humor in the story and the great illustrations, this is another winner for Leedy and students. Check it out!



Super Garage Sale—by Betsey Franco, © 2003. This book is available from ETA/Cuisenaire. A Think 'N' Share Reader, this book is fun. Three friends have a garage sale of their "junk" to raise money for some ice cream. Once the sale starts, the fun begins. This is a great story to teach students about making change and counting money. The illustrations will hold students interest as will the story. A great book for primary grades studying money.



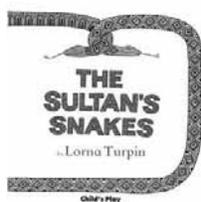
Cindy Neuschwander has done it again with her marvelous book,

Mummy Math, An Adventure in Geometry © 2005. In this book Matt and Bibi are heading to Egypt with their famous scientist parents to explore the pyramids. The story is cute, the students will like it, but I found some weaknesses that I think would make the book even better. The shapes are not labeled and the story does not present the "math" clearly. I would still use the book because it deals with a subject that students like (mummies) but don't count on it to clarify shapes for your students. The end of the book does have some activities for the students to do when they finish with the book to help with understanding the shapes.

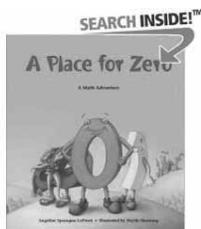


Cindy has another book on the market, this one is set in Peru and deals with patterns. Matt and Bibi use their knowledge of patterns and sequences to help them find their way around to the Lost City of Quwi. **Patterns In Peru** © 2007, begins with the children finding an ancient tunic covered in patterns and designs. The

children discover that the patterns are keys to finding the lost city. Matt and Bibi would love to find the Lost City, but they have their hands full when they're separated from their parents. Only an understanding of patterns and sequences can help them get back to their parents in this whimsical, educational adventure read. Students will love this book for the adventure as they discover how understanding patterns can be useful.



If you are looking for another book on patterns, here's one to check out. **The Sultan's Snakes** by Lorna Turpin © 1991. The sultan has lost his snake, who is hiding in the sultan's palace. There are some great patterns that will entice the students as they look for the snake.

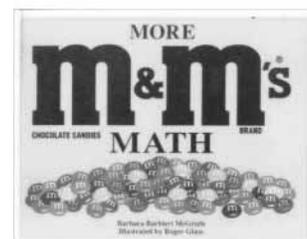


A Place for Zero, A Math Adventure, ©2003 is written by Angeline Sparagna LoPresti. Zero is sad as he tries to find his "place" among the other digits. LoPresti involves characters like Count Infinity, King Multiplus and Queen Addeleine who all live in

the land of Digitaria. As the story progresses, Zero learns of his additive and multiplicative identities, and thus, finds his "place" among the digits. Cute story, cute pictures for 2nd or 3rd grade. The pictures are a little simple for much higher than that. However, if your students like cartoon illustrations, they will enjoy the story and the math lesson.



If you are looking for some holiday fun during math time, check out Rebecca Dickinson's **The 13 Nights of Halloween**, © 1996. There are so many things going on in this book it works for nearly every grade level. Following the same format as the 12 Days of Christmas, a shy goblin is trying to woo his goblin girlfriend with different presents each night for 13 days. For younger students it's a fun story to count and/or illustrate in a counting book of their own. For older students have them calculate how many presents the goblin gave his "sweetie" during the 13 days. The illustrations are wonderful.



While this last book is not literature in the formal sense, it is a great book for building some math lessons using M & M's® chocolate candies. Barbara Barbieri McGrath's **More M & M's Math** © 1998. This book touches on estimation, graphing, problem solving, multiplication, division, and factoring. The book is illustrated with none other than those adorable M & M's spokes candies. There are lots of ways to use this book as a starter for numerous lessons, as a review, or for just plain fun. And what better way to teach math than to use the candies and then eat the lesson manipulatives.



For those of you that have used **Flatland: A Romance of Many Dimensions** © 1992 or **Flatterland** may like to know that there is now a DVD entitled **Flatland, the Movie** that is now available. While I haven't seen the whole DVD, I watched excerpts and it looks good. It's an animated version of life in a flat world.

Here's a website you can visit with more information:
<http://www.flatlandthemovie.com/>

Every Time I Teach, I Should Remember . . . : A Checklist for Every Lesson

As a new school year begins and many teachers are implementing or contemplating implementation of the Georgia Performance Standards, we all are moving more toward activity based instruction, having students actively exploring ideas and concepts, and using manipulatives. We have been urged to become less reliant on a text and more reliant on activities. I repeat the word “activity” often here because the point of this article is that teachers must be careful as they use an activity as the basis of a daily lesson.

An activity may be accompanied by a set of instructions, perhaps a rubric, and sometimes little else. How much thought have we given to what is missing in an “activity” that is essential for maximum learning to take place? I offer here a checklist of essential elements for use as you think about each day’s lesson. You might even cut out this Checklist and place it with your planning materials or in some conspicuous place to remind you that an activity is not a complete lesson . . . it needs help from you. This checklist may, in fact, be useful for every lesson we teach.

Checklist for Every Lesson

1. Is it clear to students, and to me, what they will know by the end of the lesson that they did not know at the beginning? Do I have Essential Questions or Behavioral Objectives? Will I use them to guide all of us? Will I communicate them to my students?

2. Am I employing effective teaching strategies? It is not enough to answer yes. I must be able to identify WHICH strategies I am using, and I should know that they have been shown to be effective. Are they research based? In *Classroom Instruction That Works* (ASCD, 2001), Marzano, Pickering, and Pollock identified the nine most effective teaching strategies in terms of raising student achievement. Which of those strategies am I using today? Are these strategies the appropriate ones for this topic?

3. Am I using all of the Process Standards today?

Problem Solving: Will my students have an opportunity to solve real problems today and not just do computation?

Reasoning and Proof: Will my students be required to explain answers and tell why they believe they make sense?

Communications: Will my students have a chance to talk about mathematics today with other students and with me? Will they have a chance to write about their work?

Connections: Will anything my students do today be connected to the real world? If not, I need to start over. Will I help them make connections to other areas of math and to other content areas?

Representation: Will my students have a chance to draw, graph, model, or represent in some other way problems and/or solutions?

4. How will I know my students have learned? Assessment should be formative (during the lesson; often this assessment is informal) and summative (at the conclusion of the lesson).

Formative – During the lesson, how, and how often, am I determining whether students are getting it?

Summative – At the end of class, what **evidence** will I have that they “got it.” Do I know about students individually and not just the class in general?

5. Have I planned for clear communication of expectations? If I do not plan this communication there could be chaos. Before the activity, will it be clear to students what they are to get out of the activity? During the activity, will they understand what is expected of them? After the activity, will they know what will take place and how we will close our work?

6. How will I handle instruction on this topic? Is this activity intended to apply concepts that have already been developed? If it is intended to introduce a concept, will I need direct instruction at some point or does the activity provide adequate instruction? If direct instruction is needed, when and in what form will this instruction take place?

7. How will students summarize? This item is not about a teacher summary. How will STUDENTS summarize what they have learned? Will there be summaries within the lesson? At what points will these summaries take place? How have I planned for student summaries at the end of the lesson?

8. How will students attain fluency at this skill? I must remember what coaches know - - - that to attain fluency at a skill, one must practice. How have I planned for practice to attain fluency? Practice does not have to be worksheets, but doing a problem one time may not develop fluency. Should I provide more problem-solving opportunities that use this skill or should I provide problem sets? What other ways can I promote fluency?

The ideas above are not new ideas. They are, in fact, a synthesis of what we have learned about effective teaching over the last few years. In an effort to move toward more active learning, we must not forget these basic principles that are research proven to promote higher achievement. We just need reminders and this checklist may help.

A Checklist for Every Lesson

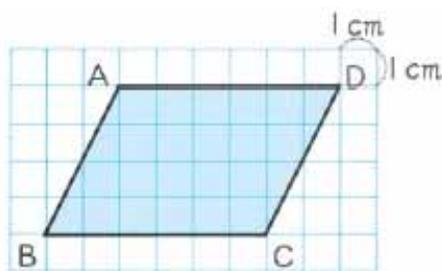
- 1. Is it clear to students, and to me, what they will know by the end of the lesson that they did not know at the beginning?
- 2. Am I employing effective teaching strategies?
- 3. Am I using all of the Process Standards today? Problem Solving, Reasoning and Proof, Communications, Connections, Representation.
- 4. How will I know my students have learned?
- 5. Have I planned for clear communication of expectations?
- 6. How will I handle instruction on this topic?
- 7. How will students summarize?
- 8. How will students attain fluency at this skill?

Deriving Area Formulae

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Deriving Parallelogram and Triangle Formulae: A Japanese Perspective

The Georgia Performance Standards M5M1 (b) and (c) state that Grade 5 students are to derive the area formulae for a parallelogram and a triangle, respectively. How might we organize our instruction so that students derive these formulae instead of teachers demonstrating their derivation? In this article, I will discuss how a Japanese textbook series (Hironaka & Sugiyama, 2006) sequences its activities to help students develop

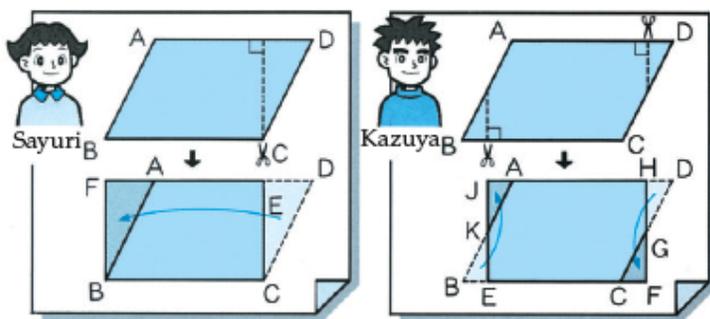


and solidify their understanding of these formulae.

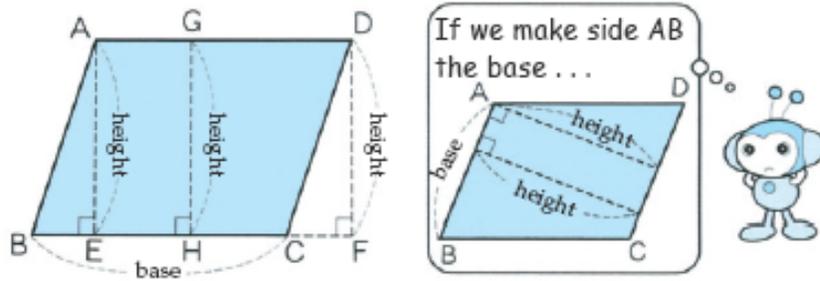
The way the formula for a parallelogram is introduced to Grade 5 students by Hironaka & Sugiyama (2006) is not unfamiliar to many of us. The first page of the unit depicts 7 different polygons (one each of a rectangle, square, triangle, parallelogram, trapezoid, rhombus, and general quadrilateral). The teacher asks the students for which of those polygons they can find the area using what they already know. This question is then followed by another question, “Can we find the area of

the other figures?” The initial activity asks students to find the area of the parallelogram shown on the left. This question is accompanied by a suggestion, “How can we change the shape into a rectangle?”

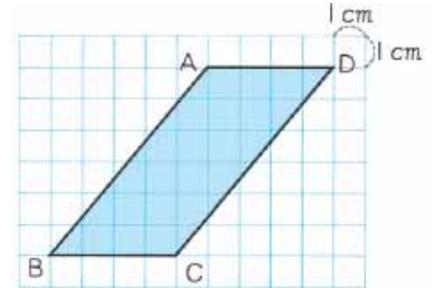
Students are then expected to use their prior knowledge to find the area of the parallelogram. They will then compare and contrast their strategies to generalize a method to calculate the area of a parallelogram. Two possible approaches are included for students to examine.



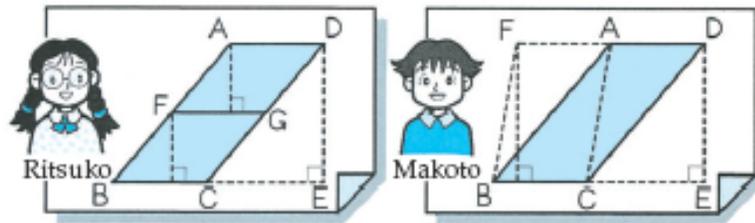
Sayuri cut and translated the right triangle DEC to form a rectangle BCEF. Kazuya, on the other hand, transformed the given parallelogram into a rectangle by cutting and rotating two smaller right triangles (BEK and DHG). Students are then asked to relate the length and the width of the new rectangle with the original parallelogram. At this point, the ideas of base and height are introduced - the base and height of a parallelogram relates to the length and width of the rectangle created by modifying the parallelogram. The textbook is very deliberate in trying to help students understand what the height of a parallelogram is in part because this is the first time students have to measure something other than the sides of a figure to determine its area. The diagrams below are included to show that the slanted side of a parallelogram may be considered as the base.



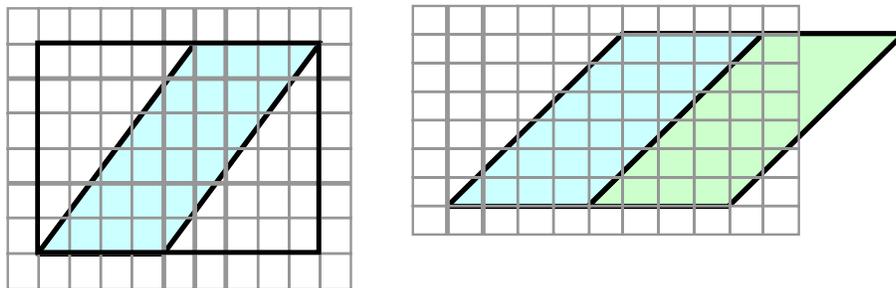
Although the textbook then defines the formula for the area of a parallelogram as (base) \times (height), it does not stop there. The next investigation asks students to find the area of the following parallelogram. Notice that, even though the parallelogram can be changed into a rectangle by using the slanted side as the base, students cannot yet determine the length of the base (nor the height) at this point. Thus, the strategy students developed in the first investigation does not appear to work in this situation. Therefore, they have to think about other ways of changing this parallelogram into a rectangle.



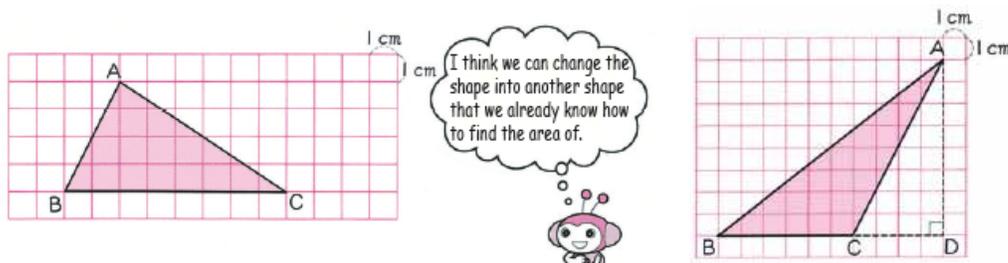
Two possible approaches are included in the textbook, but there are many other strategies students might use.



Both Ritsuko and Makoto used a familiar form of parallelogram. Ritsuko partitioned the given parallelogram into two parallelograms, while Makoto cut and moved triangle ACD to form a new parallelogram, $ACBF$. When this problem was posed to a Grade 6 students in a New Jersey public school, in addition to the two methods included in the book, several other strategies were used. Two strategies developed during the lesson are shown below. In the first method, you can find the area of the large rectangle and subtract the area of the two triangles (by noticing that these two triangles make a rectangle). In the second method, another copy of the original parallelogram was used to form the large parallelogram. The area of the large parallelogram can be determined using what they already know. Then, you can divide the result by 2 to determine the area of one parallelogram.



The formula for the area of a triangle is developed in a similar manner, except that students can now use the formula for the area of a parallelogram as well. The initial investigation asks students to find the area of the triangle on the left. Then, once the formula is developed, a follow up question asks students to find the area of the triangle on the right. Through these sequences, the textbook tries to develop and solidify students' understanding of the formulae.



When I share these ideas with teachers, there are always a few who say, “My students can’t do this.” In fact, it would not be surprising if there were students who could not determine the area of the initial parallelogram or triangle if these questions were posed to them in a vacuum. The textbook series lays the foundation for these investigations at the end of the unit on the area of rectangles and squares in Grade 4. After they study the formulae for the areas of a rectangle and a square, students investigate how to find the area of complex figures, like the L-shape shown below. The students then discuss how they can determine the area of an unfamiliar shape by changing it into something familiar. They further discuss different ways of changing a given shape into a familiar one, for example, by decomposing it into smaller parts or adding on to the original shape. Naoko and Kazuya partitioned the given shape into two rectangular regions. On the other hand, Sayuri added on a 3 cm by 5 cm rectangle to create a 9 cm by 11 cm rectangle. She then subtracted the area of the smaller rectangle from the area of the larger rectangle.

Finding the area of composite figures

2 Let's think of a way to find the area of the shape on the right!

1 Let's explain each friend's way of thinking!

Naoko Kazuya Sayuri

Some students may find the area of the given shape by cutting off a part and move it to another part to make a familiar shape. For example, Kazuya, the second child in the example below, may move the smaller rectangular piece below the larger rectangle on the left to form a 14 cm by 6 cm rectangle. Then the area of the L-shape may be determined by the multiplication, 14×6 .

Through these investigations, students are expected to understand that the area of an unfamiliar shape may be found by changing it into a familiar one (or familiar ones), and ways of making a familiar shape (or shapes) include (a) dividing the original figure into parts, (b) adding on to the original figure, and (c) cutting a part of the original figure and moving it to a different position. You may notice that these are the concepts expected in GPS M5M1(f), “Find the area of a polygon (regular and irregular) by dividing it into squares, rectangles, and/or triangles and find the sum of the areas of those shapes.” Thus, before you begin investigations of the area of a parallelogram and a triangle, it might be beneficial for students to explore complex figures composed of rectangles and squares first.

Grade 5 students can derive the area formulas for a parallelogram and a triangle themselves. However, in order to maximize the possibility that more students will indeed derive the formulas on their own, we must carefully examine the sequence of activities we pose to our students. The approach discussed in this article shows only one possible sequence. Readers are encouraged to examine the sequence used in their textbooks and other resources carefully to develop a sequence that will be most appropriate for their students.

Reference

Hironaka, H. & Sugiyama, Y. (2006). *Mathematics for Elementary School*. Tokyo: Tokyo Shoseki. (available in the US from www.globaledresources.com)

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Effective Teaching for the Development of Skill and Conceptual Understanding of Number: What is Most Effective?

Documenting which instructional methods are most effective for students' learning continues to be one of the great challenges for educational research. Should teachers use Method A or Method B?

No single study can *prove* that one method or feature of teaching is better than another for helping students achieve a particular learning goal because too many factors affect the results. But by detecting patterns across studies, especially across a set of studies that used different research designs and procedures, educators can identify robust features of teaching that seem to produce similar effects related to particular learning goals.

We select here two learning goals around which a substantial amount of data point to effective features of mathematics instruction. These goals are (1) skill efficiency—the rapid, smooth, and accurate execution of mathematical procedures (Gagne 1985)—and (2) conceptual understanding—the construction of relationships among mathematical facts, procedures, and ideas (Brownell 1935; Davis 1984; Hiebert and Carpenter 1992).

Skill Efficiency

A large set of studies conducted in the 1970s and 1980s within the

process-product paradigm (identifying relationships between what teachers do in the classroom—the process—and what students learn as a result of this instruction—the product) supplies the best evidence from which to induce patterns for links between teaching and skill efficiency. Good and Grouws (1977) examined the teaching performance of more than 100 third- and fourth-grade teachers over a two-year period. Results from the study indicated that teaching effectiveness was associated with the following behavioral clusters: whole-class instruction with demonstrations by the teacher; a task-focused environment; faster paced lessons and more homework; and classrooms relatively free of behavioral problems. Evertson and her colleagues (1980) studied junior high school teachers of mathematics ($N = 29$). More effective mathematics teachers asked more questions than less effective teachers did, with most of the questions lower-order product questions. The more effective teachers also ran well-organized classrooms focused on academic tasks and emphasized whole-class instruction with some time devoted to seat-work and practice.

Findings from a number of additional studies, summarized in Brophy and Good (1986), reinforce the following claim: mathematics teaching that facilitates skill efficiency is rapidly

paced, includes modeling by the teacher with many teacher-directed, product type of questions, and displays a smooth transition from demonstration to substantial amounts of error-free practice. The teacher plays a central role in organizing, pacing, and presenting information to meet well-defined learning goals.

Conceptual Understanding

Two features of instruction emerge from the literature as especially likely to help students develop conceptual understanding of the mathematics topic they are studying: attending explicitly to connections among facts, procedures, and ideas; and encouraging students to wrestle with the important mathematical ideas in an intentional and conscious way.

Making important mathematical relationships explicit has been shown to support students' understanding of the relationships in primary-grade arithmetic (Brownell and Moser 1949; Fuson and Briars 1990; Hiebert and Wearne 1993), middle school mathematics (Good and Grouws 1977; Good, Grouws, and Ebmeier 1983), secondary school geometry and algebra (Boaler 1998; Fawcett 1938), and college calculus (Heid 1988). The importance of this instructional feature has been found in classrooms where the teacher plays the central role of

demonstrating the relationships (Brownell and Moser 1949; Fuson and Briars 1990; Good, Grouws, and Ebmeier 1983) and in classrooms where students do more demonstrating and explaining (Fuson and Briars 1990; Fawcett 1938; Hiebert and Wearne 1993). Similar findings have been reported with instruction lasting for just days or weeks (Brownell and Moser 1949; Hiebert and Wearne 1993) and instruction lasting two to three years (Boaler 1998; Fawcett 1938).

The particular ways that important relationships are made explicit during instruction vary from study to study. In some instances, students were asked to examine carefully the differences and similarities between concrete and symbolic representatives of the same quantities and operations (Brownell and Moser 1949; Fuson and Briars 1990; Hiebert and Wearne 1993). In some situations, teachers explained in detail why the arithmetic procedures worked as they did (Brownell and Moser 1949). In other situations, students were asked to develop their own solution methods and justify their validity (Fawcett 1938; Hiebert and Wearne 1993).

It is striking that given the robustness of the link between instructional attention to important relationships and students' level of understanding, typical classrooms in the United States focus on low-level skills and rarely attend explicitly to the important mathematical relationships (Hiebert et al. 2003; National Advisory Committee on Mathematics Education 1975; Rowan, Harrison, and Hayes 2004; Stigler et al. 1999; Weiss et al. 2003).

The second feature of instruction that consistently facilitates students' conceptual understanding is the engagement of students in wrestling with, or struggling with, important mathematical ideas. We use the word

struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent.

The phenomenon of benefiting from struggling with the significant mathematical ideas can be inferred from a number of studies in which the students were presented with challenging tasks and asked to work out new solution methods on their own (Carpenter et al. 1989; Cobb et al. 1991; Fawcett 1938; Hiebert and Wearne 1993; Lampert 2001; Schoenfeld 1985; Stein, Grover, and Henningsen 1996; Stein and Lane 1996). As an example, Stein and colleagues examined the mathematical tasks presented to middle school students. Results clearly indicate that students attending schools in which teachers presented *and* faithfully implemented more challenging problems were more likely to develop increased conceptual understanding of the mathematics. Implementing "challenging" problems means that the teacher does not step in and do the mathematical work but rather allows students to do the work by struggling, if needed, to complete the problems.

Not a One-to-One Correspondence

The features of teaching we have described for promoting skill efficiency are quite different from those that promote conceptual understanding. Certainly the teacher's demonstrations, fast pacing, and error-free practice features seem to constitute a different method or system of instruction from those that present challenging problems and encourage students to wrestle or struggle with the important ideas. But this apparent correspondence between one set of features for skill efficiency and a completely different set of features for conceptual understanding breaks down when it is observed that many of the studies that focused on conceptual

development also reported that students' skills increased at a level equal to or great than those of students in the control groups (Boaler 1998; Fawcett 1938; Fuson and Briars 1990; Good, Grouws, and Ebmeier 1983; Hiebert and Wearne 1993; Stein and Lane 1996).

One way to explain this finding is that the *nature* of skill learning might be somewhat different under the two instructional systems. In one system, instruction is fast paced, teachers ask short-answer targeted questions, and students complete relatively large numbers of problems during the lesson with high success rates. This appears to yield skill efficiency. In the other system, instruction is more slowly paced, teachers ask questions that require longer responses, and students complete relatively few problems in each lesson. At least under some conditions, this appears to yield skill efficiency coupled with conceptual understanding. The cognitive mechanisms that students are likely to use in response to these different systems are different (Wittrock 1986), leading to different skill competencies.

Old Dichotomies Are Not Helpful

The features of teaching that facilitate skill efficiency and conceptual understanding do not fall neatly into categories frequently used to contrast methods of teaching, such as expository versus discovery, direct instruction versus inquiry-based teaching, student-centered versus teacher-centered teaching, and traditional versus reform-based teaching. Although the features of teaching that promote skill efficiency fit some of these labels better than others, the features of teaching that promote conceptual understanding (and perhaps skill efficiency) cut across these common labels. In particular, attending explicitly to important mathematical relationships can be done within any of these methods.

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