 Manuscript Format: Manuscripts are blind reviewed by members of the editorial review board. For this reason, each manuscript should include a cover sheet containing: title of manuscript, author’s name, position and email address. Identifying information should not appear elsewhere in the manuscript in order to ensure an impartial review.

Manuscripts should be double-spaced, with 1-inch margins on all sides, typed in 12-point font and follow the APA 5th Edition style guide. Manuscripts should be submitted in MS Word. If you have a picture or graphic in the text, please include the original picture(s) in a separate file.

Manuscript Submission: Manuscripts should be submitted to reflections@georgiasouthern.edu. Receipt of manuscripts will be acknowledged. Manuscripts are accepted for consideration with the understanding that they have not been published previously and are not being considered simultaneously for publication elsewhere. Additional inquiries should be sent to Gregory Chamblee, Editor, Georgia Southern University, Department of Teaching and Learning, PO Box 8134, Statesboro, GA 30460-8134; Phone: 912.478.5701; Fax: 912.478.0026; reflections@georgiasouthern.edu.

Manuscript Publication: When a manuscript is accepted for publication, the editor/journal reviewers may make suggestions or revisions in consultation with the principal author. However, because of publication deadlines the editor reserves the right to make minor revisions without seeking prior approval from the author. Release statements for all copyrighted materials must be received prior to publication. Upon publication, two complimentary copies of the issue are sent to the principal author.
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I'm excited to be incoming President of GCTM as we enter 2009 because it's a “magnificent time to know mathematics.” Why do I say that? Read a January 2006 *BusinessWeek* article by Stephen Baker entitled “Math Will Rock Your World” (http://www.businessweek.com/magazine/content/06_04/b3968001.htm). Here are a few key excerpts:

“But just look at where the mathematicians are now. They're helping to map out advertising campaigns, they're changing the nature of research in newsrooms and in biology labs, and they're enabling marketers to forge new one-on-one relationships with customers. As this occurs, more of the economy falls into the realm of numbers.

Says James R. Schatz, chief of the mathematics research group at the National Security Agency: “There has never been a better time to be a mathematician.”

The rise of mathematics is heating up the job market for luminary quants, especially at the Internet powerhouses where new math grads land with six-figure salaries and rich stock deals.

Top mathematicians are becoming a new global elite. It's a force of barely 5,000, by some guesstimates, but every bit as powerful as the armies of Harvard University MBAs who shook up corner suites a generation ago.

Just as mathematicians need to grapple with human quirks and mysteries, managers and entrepreneurs must bone up on mathematics. . . . “Now it’s easier for people to bamboozle someone by having analysis based on lots of data and graphs,” says Paul C. Pfleiderer, a finance professor at the Stanford Graduate School of Business. “We have to train people in business to spot a bogus argument.”

Yes, it’s a magnificent time to know math.”

Did you ever think you'd hear mathematicians called “a new global elite”? I recommend you read Stephen Baker’s 2008 book called *Numerati*. It explains how mathematics is transforming medicine, shopping, health care, politics, and work, among other areas.

The tremendous increase in computing power and ability to store data means that many companies have more data than ever before and need someone with mathematical expertise to make sense of it. What a great opportunity for our students! It also means people in most careers need more quantitative knowledge than they did even ten years ago. Our adoption of the GPS curriculum was a timely one for preparing all of our students for this world in which mathematics is more important than ever.

It’s hard to believe the end of 2008 is here already. Wasn’t it just Y2K? The end of the year is always a hectic and joyful time for me. Thanksgiving may be my favorite holiday because I spend time with my extended family at beautiful Lake Martin, Alabama. It just makes me grateful for so many things. And as the calendar speeds into December and the end of the year, my attitude moves towards one of reflection and hopes and dreams for the New Year. I’d like to ask you to take a minute to be thankful, to reflect on 2008, and to look ahead to 2009:

1) **Who are you thankful for in your personal life?**
   Who are the family and friends who support, encourage, and bring enjoyment to your life?
   I know that my life would be very different without parents who get wiser every year; an extended family who supports me even when they think I’m a nut; and friends who support, sustain, and amuse me. Friends and family ground me and keep me sane. Who does that for you?

2) **Who are you thankful for professionally?**
   Who have your mentors been?
   Take a minute to think about all those who helped you learn to teach. I know it’s taken a village to raise me as a teacher. Sir Isaac Newton acknowledged his mentors with the well-known quote: “If I have been able to see further than others, it is because I have stood on the shoulders of giants.” I don’t know about the seeing “further than others” part, but I know I stand on the shoulders of many giants. I feel fortunate to have had a
number of mentors. To be honest, without mentors at Dunwoody and Stone Mountain High, I’m not sure I would still be in education.

Who have been the giants in your professional life? How can you let them know how great an impact they’ve played in your life?

3) **What have you learned as a teacher in 2008?**

I’d like to think that, even after 24 years of teaching, I continue to improve and every year try to look back and reflect on what I’ve learned. As I look back on 2008, I know I extended my knowledge of mathematics from my work as a trainer for the GPS Math I and II curricula. I learned more mathematics from teaching a class last spring with my colleague Mary Garner that included occasional visits from mathematician Josip Derado. I learned from Mary, Josip, and the impressive group of students we taught.

What have you learned this year? In what areas have you improved as a teacher this year?

4) **What are personal goals for 2009 that get you fired up?**

Personally, I’ve learned that big goals are much more motivating than small ones. Some people have called these Big Hairy Audacious Goals (BHAGs). I’d like to challenge each of you to identify at least one BHAG in your personal life for 2009. My goal will be to do a trail run of over ten miles. I’ve run half marathons and two marathons, but have never done a trail run. What will you do? Learn to play the cello? Complete a home project? Learn a language with your child? Whatever it is, make sure it’s audacious!

5) **What are professional goals for 2009 that get you fired up?**

Remember, this also needs to be a BHAG! For my goal, it’s hard to say. I’m an interim chair this year, and I’m not sure what 2009 will bring. I’ll word my goal broadly: to learn as much as I can about leadership as a chair and to search for the right opportunity in which to apply those leadership skills. What’s your audacious goal for your teaching? You’ll be surprised at the power of a true BHAG.

That’s your “homework” until the next issue. Email (lstallin@kennesaw.edu) me and let me know your goals so I can be excited with you too. Thanks for taking the time to read this column at a very busy time of year. Have a wonderful Thanksgiving, holiday season, and an exciting 2009!

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**Call for Reviewers**

The journal is in need of reviewers. If you have an interest in reviewing please send your name to reflections@georgiasouthern.edu.
May I introduce you to ON-LINE membership maintenance:

If you are a new member of GCTM or have registered for the Georgia Mathematics Conference, you have probably noticed the new look of the GCTM.org site. It is now possible to renew and update membership information online, you can now pay by credit card, PayPal, or by sending a cheque or money order to the Membership Director using the membership form in this issue.

Here are some suggestions and information as you use the system for the first time.

- If you have an email address on file with GCTM, your membership record should be in the online database.
- The first time you sign into the GCTM.org website, you should verify your personal information. You will be allowed to enter a password.
- Your record is recognized by your email address. If you are denied access, you might try any other email address you might have used in the past. You can then change this information as needed.
- Your membership number will have changed from the one you were issued by GCTM.
- Middle names are not required, but I encourage you to at least use a middle initial. We do have duplicate names and the middle name is helpful as an identifier.
- If you do not have an email address, it would be very helpful if you could sign up for an address at one of the many free sites.
- If you experience any problems, please contact me at scraig@gctm.org, or at my phone or mailing address.

Thank you for your patience as we work to make membership easier and more accessible. As with any change, it will take a little time to work out any glitches. Contact me if I can help you in any way.

The Executive Board passed a change to membership requirements at the recent Board meeting. They agreed to offer a $10 membership level for full-time graduate students who have taught before, but who are not currently teaching. We are hopeful that these changes will make it easier to renew and be a member of GCTM and we will see many new and renewing members!

Membership remains at the same level of about 2,500 members. Encourage your colleagues to join and lapsed members to renew, GCTM, the best bargain in town!
Have you completed your PLU credit that you began at the Georgia Mathematics Conference in October?

If you attended at least 10 hours of sessions at the October 2008 conference at Rock Eagle, then you need to complete steps 7 and 8 as soon as possible. The PLU Course completion form is on the GCTM website if you have misplaced your copy.

STEP 7: Return to your school or workplace and begin to implement some of the strategies or ideas that you learned at the conference. You must do both of the following:

1. Schedule a “sharing” session to share strategies and ideas that you learned at the conference. This session should be appropriate to your responsibilities in your workplace.
2. Schedule a “classroom observation” or “model teaching session” that demonstrates strategies or ideas learned at the conference. Someone authorized to make this observation should conduct this observation and he/she must verify its quality. This person must sign the PLU Course Completion form.

STEP 8: Following the completion of the sharing session and the classroom observation, the participant is responsible for returning the signed completion form (signed by the system-designated person i.e., Principal, Supervisor, etc.) to the Georgia Council of Teachers of Mathematics (GCTM).

The PLU Course Completion Form should be completed, signed and returned by Feb. 1, 2009 to:

Becky King
Executive Director, GCTM
5314 Brooke Ridge Drive
Dunwoody, GA 30338-3127

STEP 9: Completed verification of your PLU credit will be returned to you. It is your responsibility to send this PLU credit form to your accrediting agency when needed.

GCTM will match and verify that all activities and artifacts (prior approval form, training and completion forms) are received and in order. GCTM will return a certified course completion form to the participant.

It is the participant’s responsibility to submit the final documentation to the certifying agency.
Join NCTM Today

Today is a golden opportunity for you to become a member of the National Council of Teachers of Mathematics. Why should you join NCTM?

- The National Council of Teachers of Mathematics is a public voice of mathematics education, providing vision, leadership and professional development to support teachers in ensuring equitable mathematics learning of the highest quality for all students.
- A great way to network and keep abreast as to what is going on in mathematics education.
- NCTM is dedicated to providing professional development for teachers to ensure mathematics learning of the highest quality for all students. They have many on-line courses and events that can help you grow and learn while earning staff development hours.
- NCTM has wonderful online prepared lessons that you can copy and use in your classes.
- You can purchase books at a member discount.

Just follow this link: http://www.nctm.org/membership to find out how to become a member. There are individual memberships and other types of memberships available.

Join today! You will open your learning opportunities and find an abundant supply of really top-notch thinking resources for your classroom.

I know as you read this column the elections are over; however, listed below are some of the election resources at NCTM.org you can use in the future. Everyone can find fun and exciting activities for his/her classroom; no matter what level you are!

**Election Resources**

**Elementary**

- **ELECTING A PRESIDENT** - Short activities sorted by K-2, 3-4, and 5-6 *(Teaching Children Mathematics)*

**Middle School**

- **PRESIDENTIAL PLAYING FIELD** - Lesson exploring electoral votes from 1888 to 2000. *(Mathematics Teaching in the Middle School)*
- **GETTING INTO THE ELECTORAL COLLEGE** - Unit exploring the electoral college, focusing on percentages, ratios, area, problem-solving and reasoning skills. *(Illuminations)*
- **WHAT PERCENTAGE DOES IT TAKE TO WIN A VOTE?** - Exploring elections and percents in a school setting. *(Figure This!)*

**High School**

- **PREDICTING THE PRESIDENTIAL ELECTION** - Least-squares linear regression lesson to predict the 2004 presidential election. *(Mathematics Teacher)*
- **HOW MANY VOTES?** - Mathematical modeling lesson to determine the fewest number of votes necessary in order to be elected president. *(Mathematics Teacher)*
- **WILL THE BEST CANDIDATE WIN?** - Lesson in which students explore advantages and disadvantages of alternative voting methods. *(Mathematics Teacher)*
- **MINIMUM FRACTION OF THE POPULAR VOTE TO ELECT THE PRESIDENT** - Polya’s general solution and discussion of implications in high school applications. *(Mathematics Teacher)*

- **GETTING INTO THE ELECTORAL COLLEGE** - Unit exploring the electoral college, focusing on percentages, ratios, area, problem-solving and reasoning skills. *(Illuminations)*
Online Tools

State Data Map - Interactive US map that shades the states proportionately according to population, electoral votes, or data the user inputs. (Illuminations)

Electoral Calculator - Predict who will win the next election by entering which party will win each state. (National Archives)

Math and Voting - See how your vote influences the outcome of the election using different methods. (American Statistical Association)

General Information - Information about elections in the US, other voting methods, and the math of voting methods. (NCTM)

Call for Manuscripts

Topics:
GPS implementation manuscripts are needed. For example, instructional strategies to teach GPS, GPS implementation issues, working with special populations in a GPS environment and sample student task solutions are some of the ideas of interest.

Teaching Tips Ideas:
Share with your fellow teachers a pearl of instruction or assessment wisdom you have used in your classroom. Topics include how to design and implement effective warm-ups, strategies for implementing journal writing, etc. Manuscripts published in this section are typically one page in length.
A new tradition began in 2008 when the Georgia ARML (American Regions Math League) didn’t have to take a long and tiring bus ride to Penn State to compete in their annual math tournament. In 2008, the competition was held, for the first time, at the University of Georgia.

All through the year, the ARML coaches watch students at math tournaments and look for what might be the top high school math students in the state. They encourage all students to take the National Math Competition given in February so they can get a good comparison of students on a national level. Although most of these top students are part of a team that will be invited to the state competition, the coaches make sure that any math student with the potential to be one of the top 30 in the state also attends the state competition just to possibly qualify for the ARML team. At the state competition, the coaches use the State Tournament individual ranking, the AMC/AIME results, scores from other tournaments during the year, and past ARML experience to select 33 students for the ARML team. Since only 15 students compose an ARML team, the first 15 students selected are expected to form the A team and the next 18 are selected for their growth potential. However, students don’t know the order in which they are selected, so they jockey for position on either the A or B team throughout the practices.

This past year, ARML coaches Tom Fulton, Chuck Garner, Ben Hedrick, Adam Marcus, Debbie Poss, Steve Sigur, and Don Slater selected 33 students to be on the two ARML teams, and then let other interested students join the team by covering the cost of their expenses. (GCTM basically funds two teams.) In the end, 41 students attended the competition.

After the team was selected in April, they met each Sunday at one of the coach’s schools to practice. Practice includes preparing students to compete in each of the 4 divisions of the competition: team round, power question, individual round and relays. During the team round, all 15 students work together to solve 10 problems in 20 minutes. They may use calculators and may collaborate any way they see fit, but at the end of the 20 minutes, only the answers to the 15 questions are graded. During the power round, the team receives an “interesting situation” that usually depends on a crucial underlying mathematical concept. They have one hour to work through questions relating to that situation, and write up proofs of various relationships relating to the situation. During the individual round, each student is given two problems, and must work alone to solve both of them in less than 10 minutes. In the relay round, students divide into 5 three-person relay teams. The first person receives a question, must work it and pass the answer to the second person on the team who uses that number to answer his question. He passes his answer back to the third person who uses that number to answer his question and finally turns in a final answer to be graded.

The big change occurred on the Thursday before the actual competition. Instead of boarding a bus for a 2 day bus trip to Penn State, students met after lunch at UGA. They checked into the dorms and practiced until supper. After supper, they practiced a little more and after some recreational activities, settled into their rooms. The next morning, they ran through a complete ARML practice day – staying as close as possible to the actual locations and using the exact time frame. At that point, other states were arriving, so students had free time to meet other students and relax. Many of them chose to practice more after supper, even though several activities had been planned for the mathletes. The preparation paid off, because students knew exactly where to be and when to be there for the actual competition. All they had to worry about was the math…

At the end of the day, they came in 7th place in the nation and, although somewhat upset that they hadn’t placed higher, were happy to go celebrate the experience.
Forty-one students from all over Georgia traveled to the University of Georgia to compete in a national math tournament late last May. They were selected to be a part of the Georgia All-State Math Team competing in the 33rd annual American Regions Mathematics League (ARML) competition, held simultaneously at four locations on May 31: Penn State University, the University of Iowa, the University of Nevada at Las Vegas, as well as at the new southern site at the University of Georgia. The ARML tournament brings together the most mathematically talented students for the largest on-site mathematics competition in the nation.

The Georgia A Team placed 7th out of over 120 teams competing this year. The Georgia B Team, comprised primarily of underclassmen, finished 29th overall. Teams of 15 students and 3 coaches, representing states from across the United States, Canada, Taiwan, and the Philippines practiced for several months for this intense competition.

Three of the four parts of this meet involve teamwork such as the Power Question, which is a one-hour challenge where each team writes solutions and proofs to a complex problem. This year's topic was based on tiling patterns, in which students had to prove how and why it works as it does. Other parts of the competition include a Team Round where the team is given 20 minutes to answer 10 questions, and a Relay Round where students pass their answers on to teammates who need it to solve their problems. The competition also features an Individual Round where each student independently solves eight difficult questions against competitors.

The Georgia teams were partially sponsored by the Georgia Council of Teachers of Mathematics. National sponsorship is primarily provided by the D. E. Shaw group, a specialized investment and technology development firm.

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**GCTM Grants and Awards Opportunities**

- Gladys M. Thomason Award for Distinguished Service
- Dwight Love Award
- John Neff Award
- Awards for Excellence in the Teaching of Mathematics (Elementary, Middle & Secondary levels)
- Teacher of Promise Award
- Mini-Grants
- Special Projects

For additional information visit the GCTM website [www.gctm.org](http://www.gctm.org).
The Southwest Region was well represented at the 2008 Annual GCTM Conference at Rock Eagle held this year on October 15, 16, and 17. Many members from all over the region were in attendance and were able to enjoy several of the mini-sessions, vendor displays, informational and inspirational meetings, as well as the Southwest Regional Caucus.

In addition, the Southwest Region had members presenting at the conference to share their many talents with others. Two of our talented ladies presenting at the conference were Mrs. Sherry Ammons and Mrs. Debbie Hamlin. Both teach at Stewart County Elementary School. Mrs. Ammons demonstrated techniques and procedures to keep a meaningful “Math Journal”. Mrs. Hamlin shared a holiday activity, “The Twelve Days of Halloween” for math that was done to the tune of “The Twelve Days of Christmas”.

Those attending the caucus had a great time practicing our “cheer”, making the banner, and getting organized (?) for the rally! Everyone was glad to have Dr. Lettie Watford, Vice-President for Advocacy for the GCTM and Dean of the School of Education at Georgia Southwestern University, along with Dr. Chu Chu Wu, professor in the School of Education at Georgia Southwestern State University, joining us and participating. Thanks to everyone who attended and helped make Southwest Region SHINE!

Many of you may remember the article on “Mathfest” for the Southwest Region last year. Well due to copyright, the event will now be called “Math Menagerie”. This year, “Math Menagerie” will take place at Macon County Elementary School in Oglethorpe, GA. Confirmations have already been received from schools in Sumter and Lee Counties. Several more systems have been invited, and we are awaiting their reply at the writing of this article.
The deadline is not until December 15. The event is for third, fourth, and fifth grade students to compete for prizes, trophies, and medals. All problems used in competition are based on Georgia Performance Standards. As soon as schools are registered, they are sent packages that include competition rules, practice problems, and directions with itineraries. Each participating school may bring 15 students from each grade (total of 45 on each team) per school. Should a school so desire, they may bring more than one team. The registration for each team is only $50. Anyone wishing to start a “Math Menagerie” event at their school may contact Dr. John H. Walker, Sr. at 478-472-7221 Ext. 3412 or 229-938-0640 after 5:00 pm; walkerjsr@yahoo.com; or 318 Paschal Street; Plains, GA 31780-5668. Southwest Region members are always glad to share their ideas and talents in promoting mathematics!
Retirement Concern

William I. (Bill) Marsh (retired in 2002)
GCTM Life Member
Central-West Region Representative for 15 years
P.O. Box 335
Fayetteville, GA 30214
bnmarsh335@comcast.net

Retired GCTM Members and Those Nearing Retirement:

The Teachers Retirement System of Georgia is one of the most financially stable and expertly managed retirement systems of its kind in the United States. As a retiree or one nearing retirement this is vital to your future!

You should be aware that TRS has automatically given retired educators a 1.5% Cost of Living Adjustment (COLA) every six months since 1969. It is part of the written policies of TRS! Governor Purdue has asked the TRS Trustees to change the policies so that the Trustees must vote every six months to pay the 1.5% COLA. The Trustees will or have decided that issue on November 19, 2008.

Have you considered joining the Georgia Retired Educators Association (GREA)? Membership in GREA will keep you informed about TRS and your benefits. GREA has, as one of its principle purposes, the financial interests of retired educators in Georgia. GREA is the only association to lobby the Georgia Legislature solely on behalf of retired educators.

Please consider joining and participating with the Georgia Retired Educators Association (GREA) and its local units throughout Georgia. Learn more about GREA at www.garetirededucators.org. Write them at GREA, 615-C Oak Street, Gainesville, GA 30501-8522.

Learn More About Your Organization

www.gctm.org

* Grants and Award Information
* Membership Renewal
* Mathematics Competitions
* Previous Reflections Issues
* Other
When they left my class yesterday, they knew how to factor a trinomial. What happened? “When class was over yesterday, every one of them could divide fractions. Not today!” “On Friday, they knew all of their basic decimal/fraction equivalents. By Monday, you would think 80% of them were absent when I taught it!”

What did happen? Or more to the point, what didn’t happen? The answer is that the knowledge was in their working memory, but it did not make it into long-term memory. You probably had a great activating strategy, so you caught your students’ attention. You had a variety of activities, so students stayed on task during the lesson. And - - - they LEARNED the mathematics. You saw them work problems on their own. You gave an assessment at the end of class and they could work the problems. What didn’t happen? They didn’t retain the knowledge.

Long term memory is generally defined as retaining knowledge after at least 24 hours. That quick quiz at the end of class, that closing summary in pairs, that ticket out the door - - - those assessments were testing working memory, but not long term memory. David Sousa, in his book, How the Brain Learns (2001), provides much information that is pertinent to understanding teaching and learning.

By the way, what do these findings tell us about assessment? Allowing students to review, or the teacher conducting a review, at the beginning of a period before a test does not allow us to measure what students retained. You are just measuring what they can put in working memory for a few minutes, so those assessments are completely invalid. All of the stories about students cramming for an exam and then not remembering the material afterward are not just stories. That is what really happens in the brain, and that procedure is simply wasting everyone’s time.

Classroom performance does not guarantee storage and retention. Here are some quick examples that illustrate what happened:

- You look up a phone number and learn it for the time it takes to dial, but that number does not go into long-term storage, so tomorrow you won’t remember that number.
- When you took a course that involved using a spreadsheet, you learned how to create a graph in class, and could create one on your own. Then when you got home - - -.

You did actually learn these things for a time, but they never made it out of working memory and into long-term memory - - exactly what happened to your students. What science tells us about the brain and how it learns is that to make it much more likely that learning is retained, two things must happen: The learning must make sense and it must have meaning. Though those items sound similar, they are two different things.

Brain scans have shown that when learning can be comprehended (makes sense) and can be connected to the learner’s experiences (has meaning), cerebral activity is significantly increased and retention is dramatically improved. Let’s examine making sense and having meaning in mathematics.

Making Sense

First, does the mathematics make sense? Today, mathematics teachers do a better and better job of helping students make sense of learning. We have learned how to use number rods of ones, tens, and hundreds to help students understand regrouping. We have gotten really good at helping students learn the algorithm for dividing mixed numbers. We use snap-together cubes to let students model and make sense of addition and subtraction. We have
created boxes or drawings to help students become proficient at factoring trinomials. The actual methods make sense to students and they can follow them, but one day later, those activities and hints and methods may be lost. Making sense is important, but it is not enough.

It turns out that meeting the meaning criteria for entering long-term storage in the brain is the one most ignored by teachers. We spend about 90% of our time devising lessons so that students will understand the idea (i.e. make sense of it), but to convince a learner’s brain to retain the idea, we need to help establish meaning. Making sense is important, but it is not enough. In mathematical terms, making sense is a necessary but not sufficient condition for retention.

Making Meaning and Making Memories

Meaning is personal. Is this learning personally relevant to the learners? How does it connect to their reality? For what purpose should they learn this? Here are some examples of making sense and meaning in mathematics:

- A Hershey bar is divided into 4 equal pieces and a child gets one of those pieces. This division makes sense of the symbolic representation \( \frac{1}{4} \) for the idea of one out of four. This idea has meaning because the child understands the idea of sharing a candy bar among 4 people.

- A student studying parabolas makes sense of the shape by watching a tennis ball bounce. She can build a time/height graph, and can make sense of the x intercepts as the times when the ball hits the floor. This idea has meaning because the tennis ball hitting the floor is a personally meaningful example of an x axis and intercepts. It fits with her understanding of one thing intercepting another.

The Georgia Performance Standards and the NCTM Standards have a name for making meaning of learning. We call it the Connections standard, and we know that we need to connect the mathematics we are teaching to our students’ real worlds, to other disciplines, and to other mathematics. But we get caught up in the part of effective instruction that is sense-making and lose sight of making meaning.

Does it matter to you whether students remember the mathematics you have worked so hard to share with them in a class? If it does matter, I suggest that you give serious thought in your planning every day to how you can make meaning of the mathematics. One really good way, along with connections to real life, is simply to ask the question, “What does this mean?” over and over and over and over. What does this mean in terms of other mathematics? What does this mean as related to another content area? What does this mean in terms of the problem we just solved?

If the answer to a problem is \( x = 6 \), what does this mean? Students must be able to tell you it means that letting the variable \( x \) have the value 6 will make that equation true.

If the solution to a problem about dumptrucks is 7, what does this answer mean? Students must be able to tell you it means that it will take 7 dumptrucks to move all of the dirt at once.

If the answer to simplifying an exponential expression is \( a^8b^{14} \), what does this answer mean? It means that a simpler way to write \( (a^7b^7)^2 \) is \( a^{14}b^{14} \).

If students added 136 and 157 to get 293, they should be able to give an example of what that could mean, such as the following: It means that if I have $136 and I get $157 more dollars, I will have $293 all together.

We must also remember that it does not help students make meaning if we continually tell them what the answer means. They must be able to communicate (another standard, as you know) that mathematics to you and to other students. Our job is to provide that opportunity.

Write in your plans how you will make meaningful connections, make opportunities for students to form those meanings and verbalize them, and in the process, you will improve the likelihood that the mathematics moves from working memory to long-term memory. Make meaning and you make memories.

Reference

As I was selecting books for this column, I noticed a common theme running through many of them—food. Maybe it’s because it’s late afternoon and I’m hungry or maybe it’s because students love it when they can eat their “manipulatives”. Whatever it is, I think you will find these books make great springboards to your lessons.

The first book I wanted to share with you makes a great introduction to fractions, by way of an apple. *Apple Fractions* is a Rookie Read-About Math book by Donna Townsend. (©2004, Scholastic Children’s Press.) This little book introduces students to fractions with very clear pictures and simple text. It tells and shows them examples of 1/2, 1/3, and 1/4. The book has a simple glossary of new words. Not only do students see a fraction of an object but it illustrates a fraction of a set. This book would make a great extension of a lesson or introduce fractions to little ones. You could make it even more visual by using real apples and a plate of four muffins to “act out” the different examples.

*Gator Pie* by Louise Mathews (©1979, Putnam Publishing, 2004 Sundance Publishing) is another book that includes food and fractions. This book has been around for a long time but is still one of my very favorite stories, not only because of the adorable story but for the fact that it teaches a hard concept for many students to grasp.

This book is about Alice and Alvin Alligator who find a pie in the woods and decide to share it. Before they can cut the pie, more and more animals show up and want some of the pie. The numbers grow until Alice counts 100 alligators that ALL want some of the pie. The alligators get into a fight over the pie and before it’s over with, they are all fighting so Alice and Alvin grab the pie and run off. Then they each get fifty very tiny slices or half a pie. The illustrations are great, the story is funny and students see that the larger the denominator, the smaller the piece. The book is often hard to find, but if you can get it, do so. You won’t be sorry.

While the title of this next book refers to food, there’s none used for the lesson. Nonetheless, it is a great book for teaching area and perimeter. Marilyn Burns’ *Spaghetti and Meatballs for All* (© 1997, Scholastic Press) is part of the Marilyn Burns Brainy Day Books. The Comforts are planning a family reunion and trying to decide the best way to arrange the tables so everyone has a seat. The story is fun, the illustrations are very amusing and the children will love it. Learning about area and perimeter is fun in this lesson. You will also find several pages of additional activities you can do that are related to the book.

*For those students working on their multiplication facts,* check out Jerry Pallotta’s *Hershey’s Milk Chocolate Multiplication Book* (2002, Cartwheel Books, Scholastic Inc.). This book is illustrated with a mix of photos and drawings, making the book even more appealing and inviting. The story lends itself to the use of real candy. The book uses the small segments of a Hershey bar to demonstrate how multiplication arrays are constructed. Younger students can use this book to see the patterns in repeated addition. Any level can use the Hershey bars as the basis of some “delicious” problem solving.
that all children think about at Halloween and that’s candy. And candy can lead to sorting, patterning, weighing, and attribute lessons. This book makes a great launch lesson for graphing too. You could give each student or pair of students a small bag of candy. Let them see how many ways they can sort the candies (by size, shape, color, texture, wrappers, etc) and then have them write about or illustrate the sets they created. Have them develop a graph showing how many candies of each type they have. They can also create various patterns with the candies. Older students can either make graphs or determine percentages or ratios to the whole set.

Now, this book has nothing to do with eating, but it does have to do with Halloween. If you like The Twelve Days of Christmas then you will love Rebecca Dickinson (1996, Scholastic Books Inc). Here’s the story of a shy little goblin that wants to impress his sweetie. The story is written in rhyme, making it appealing to younger children. The illustrations are delightful and very colorful. The framework of the story invites students to try their hand at creating their own version of the book. Primary students could draw what the goblin gave his sweetie each day and then write the corresponding number sentence (or just count them). Older students could add all the gifts for a total or write an equation for calculating the total, even if ghostie carried on this gift giving for some additional days. Whatever you do with the book, check it out for some “bewitchingly” fun math lessons.

A another good book to read if you are working on repeated addition or multiplication is a story by Inga Moore. Six Dinner Sid (1991, Simon and Shuster). Sid, a friendly neighborhood cat lives and eats at Number 6. But he also lives and eats at Numbers 1, 2, 3, 4, and 5 as well. But only Sid knows this until something happens. Children love this book and love to see what Sid will do next. Here’s a great book for repeated addition or multiplication. Let younger students illustrate the events in Sid’s live and older students use the story to develop multiplication sentences and write problem solving scenarios. If you are working on multiplication facts, try having the class write a story about “Five Dinner Fred” or “Four Dinner Felix”, depending on the fact table you are emphasizing. Bind the stories and let students read them in your class library.

This next book has been around for quite a while but it’s one that makes every child laugh out loud. Moira’s Birthday was written by Robert Munsch (1992, Annick Press, Ltd.).

It’s Moira’s birthday and she invites Grades 1-6 to her party. This sends her parents in a state of shock as they try to figure out how much food to order and where to put it, how to clean it up, etc. Each page of the story brings a new situation to which Moira has a simple solution. While the story itself appeals to all elementary students, the problem solving scenarios that can be developed from this story are more suited to middle to upper elementary. This is a story that is best to read to your class once just for fun. Then go back and reread it to them for the math. This is another book that lends itself to student writing as they create their own birthday party.

If you are working on percentages, fractions, or decimals, then you need to read Twizzlers Percentages Book by Jerry Pallotta (2001, Scholastic, Inc.). The book reviews operational signs and then uses Twizzler sticks to illustrate 100%. The book also reviews place value, decimals, and fractional equivalents. The illustrations make it easier for students to visualize the numbers. This is ideal for upper grade and middle school math students who need to see the math.

So many books, so much math, how do you decide which to choose. I’ll be back next time with more book reviews and ideas from Judy’s Book Shelf: If there’s anything you would like more information about, just e-mail me and I’ll try to help you out.
The importance of problem solving has been emphasized for decades. Two recent reports have dramatically reemphasized this goal. The National Council of Supervisors of Mathematics in a position paper on basic mathematical skills published in January, 1977, states the following:

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. . . . Problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

More recently the National Council of Teachers of Mathematics in its booklet entitled An Agenda for Action: Recommendations for School Mathematics of the 1980s strongly recommends the “problem solving be the focus of school mathematics in the 1980s.”

These are only a few of the many voices exhorting us as classroom teachers to better help our students learn to solve problems. Perhaps our position is humorously presented in the following piece of poetry called “Grooks.” “We will have to produce problem solvers galor/for each problem solved creates 10 problems more.”

How Are We Doing?

Of course we all know that teachers have been attempting to teach children to solve problems for many decades. What are the results? A recent report on the National Assessment of Educational Progress in Mathematics (completed in 1978) stated the following:

If it were necessary to single out one area that demands urgent attention, it would clearly be problem solving . . . . At all age levels, there appeared to be little attempt to think through a problem to arrive at a reasonable answer.

It also appears that the overall decline (In 1978 as compared to 1973) is greatest at the application or problem solving level (e.g., in 1978, 28% of the 9 year olds solved a simple word problem involving multiplication, compared with 46% in 1973).

What Is a Problem?

In order to improve our success rate in helping children learn to solve problems, we might first review our idea of what a problem is. Consider the following three situations.

1. Lisa bought 4 notebooks. Each notebook cost 72 cents. How much did she pay?
2. How many squares can you find?
3. 376 x 48 =

As you view these situations, ask yourself which of these is, by your definition, a problem for a third grade student. Make your choices and then check the following oft-stated definition of a problem to see if it agrees with your definition.

A real problem for an individual is a question which:

1. Presents a challenge which
2. Cannot be resolved by some routine procedure known to the individual and where
3. The individual accepts the challenge!

Standard Problems Versus Process Problems

Much has been said recently about exposing children to process problems and reevaluating the emphasis placed on standard problems. Some persons think of process
problems as problems involving a higher level of thinking—problems not simply solvable by choosing and using a particular operation. The following examples may help provide a better perception of the difference between these two types of problems.

1. Standard Problem
Ronald’s family plans to rent a camping trailer. The rent is $18.75 a day. What will it cost to rent the camping trailer for 14 days?

2. Process Problem
Joe and Tina are playing a game. At the end of each game, the loser gives the winner a chip. After a while, Joe has won 3 games and Tina has 3 more chips than she did when she began. How many games did they play?

It seems safe to say that process problems are simply problems which are more difficult for the average child and which require considerably more logical reasoning. In thinking of process problems a teacher is reminded of Stockmayer’s Theorem: “If it looks easy, it’s tough. If it looks tough, it’s durn well impossible.”

Helping Children Learn to Solve Problems

The following problem solving strategy is a simplified general look at the procedure involved in solving problems.

1. Read carefully to find the facts.
2. Look for the question.
3. Decide what to do.
4. Find the answer.
5. Read again. Does your answer make sense?

It is interesting to note that Step 4 involves the computational aspects of problem solving, that is, our skills with algorithms specifically help us complete phase 4 of this problem-solving strategy. Once this is clearly understood, a crucial question arises, how do we help children learn how to do Step 3? That is, how do we help children learn how to “decide what to do”? This is obviously the crucial facet of problem solving. The following diagram helps put problem solving in perspective. We have spent a great deal of time helping children learn how to “determine the answer.” We have spent much less time helping them learn how to “decide what to do.”

The Two Faces of Problem Solving

In order to be able to help children learn how to “decide what to do,” teachers must put problem solving in perspective. We must realize that problem solving goes hand in hand with important other learnings in the classroom. Specifically, a good problem solver must have:

1. A clear understanding of the basic concepts.
2. An ability to see patterns and form generalizations.
3. A working knowledge of basic facts and algorithm skills, and
4. A confident, patient, enthusiastic, etc., attitude toward solving problems.

As we attempt to better help children learn to solve problems we must seriously ask ourselves the question, “What shall we reward?” John Holt made a striking observation when he said, “The classroom has become the temple of workshop for the right answer. The way to get ahead is to lay plenty of them on the altar.” Clearly, the views children have about problem solving are shaped by what we reward every day in our classrooms. At a minimum, we should reward,

1. Thinking, reasoning.
2. Educated guessing—with follow-up
3. Idea-getting
4. Patience, perseverance
5. Searching for patterns
6. Flexibility

It is okay to make remarks such as “Jane—I see you’re thinking. That’s a nice discovery or, “Joe’s really taking time to think through this problem.” We want students to (1) take risks—just “try something,” (2) sick to it—keep trying after you think you can’t do it, (3) don’t hurry—allow lots of time for ideas to come, (4) be flexible—if one path doesn’t reach success, try another.

Some Specific Problem-Solving Suggestions for Teachers

To help children better learn to solve problems, the teacher is encouraged to:
1. Pose problems at all levels. The following problem can be used with very young children and can be made more complicated for older children.

**The "Pen Pals" Problems!**

Draw 3 fences (lines) so that each animal is alone in a pen.

2. Include all types of problems. It has often been said that variety is the spice of problem solving. The following types of problems suggest that this is true. All types of problems have a place in the elementary school curriculum.

   a. Puzzle Problems
   
   10 cents to cut a ring
   20 cents to weld it back together.
   How much to make the cheapest chain?

   b. Geometric Problems
   
   How far is it from home place to 2nd base?

   c. Real Word Problems
   
   Give the prices of 3 different lunch choices you would have which cost less than $2.00 (no tax):

   - Hamburger: 85 cents
   - Drink: 35 cents
   - Fries: 45 cents
   - Salad: 95 cents
   - Hot dog: 65 cents
   - Shake: 75 cents
   - Soup: 55 cents

   d. Open Ended Problems
   
   In how many ways can you measure a ball?

   e. Problems for a Calculator
   
   A diesel auto cost $2400 more than a standard model.

   Gas and maintenance costs:
   - Diesel: 1.5 cents/km.
   - Standard: 4.5 cents/km.

   How far would you have to drive for the diesel to be the most economical buy?

   f. Project Problems
   
   What will it cost for grass seed for the school ground—not counting sidewalks and driveways.

   Make plans for redecorating a bedroom: how much will it cost?

3. Have children invent problems. The following problems is said by Clifton Fadiman to have been invented by a third grade student who was asked to make up a problem like the one seen in a textbook. It is important for children to create problems from a theme given by the teacher or simply a problem to fit a number sentence.

   My father is 40 years old. My dog is 8. If my dog was a human being, he would be 56 years old. How old would my father be if he was a dog? How old would my father plus my dog be if they were both human beings?

4. Put text problems in perspective. Many of the problems labeled “For Fun,” “Think,” “Extra for Experts,” etc., in textbooks are essentially process type problems. Often teachers omit these problems because they think they’re too hard for the average child. Many children can solve these problems if selective hints and teacher help are given.

5. Put a premium on thinking. As indicated earlier, process problems can and should play a significant role in elementary school. The following problem is much more easily solved when the child realizes that it is possible to start with three balls on each side of the balance.

   8 balls all look alike. One of them is heavier. All the other balls weigh the same. How can you find the heavier ball using the balance only two times?

6. Post and push problem-solving strategies. The following is a more detailed list of suggestions which can be used along with the more general problem-solving strategy presented earlier. You should look for ways to help children have these kinds of experiences when learning to solve problems.

   a. Understand the problem.
   
   Read it (orally)?
   Restate it.
   Dramatize it.
   Identify given and needed information.
b. Try to solve the problem.
Brainstorm for ideas.
Draw a diagram, use a model or a graph.
Look at it another way.
Use guess and check.
Make a table, look for a pattern.
Find a simpler, similar problem.
Write an equation.
Work backwards.
Try your best ideas.

c. Complete any necessary calculations.

d. Check the solution.
Check calculations.
Estimate answer. Are you close?

e. Follow-up.
Study the solution process.
Find another solution.
Make up a related problem.

7. Use “thinking games’ often. Some thinking games which interest students and provide avenues for the type of reasoning involved in higher level problem solving are Kalah, Score Four, Master Mind, Attribute Games, Nim, Othello, Pico Fumi. Many of these games are available commercially or have been programmed for microcomputers. Homemade versions can also be produced.

8. Use estimation not only to check the reasonableness of answers to problems, but to help motivate children to become involved in the problem-solving process.

9. Provide lots of problem-solving practice. Here are some more problems you may wish to try with your students.

a. You have lots of pennies, nickels and dimes. In how many ways can you pay for a 15 cent whistle?

b. A multiple of eleven I be, not odd, but even, you see. My digits (a pair) when multiplied there, make a cube and a square out of me. Who am I?

c. If letters are worth these amounts:

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How many 1 dollar words can you find:

lightning  squares  courses  imported
telescope  rhombus  turkey  mittens
highways  percents  pumpkins  specials
nosebleeds  quarters  quills  settled
elephants  clockwise  dragon  settles

A farmer passed away and left 17 mules to his three children. The will specified that the oldest was to get $\frac{1}{2}$, the second eldest $\frac{1}{3}$ and the youngest $\frac{1}{9}$. They couldn’t solve this problem by dividing up a mule, and an argument ensued.

A friend, riding by on his mule, stopped and immediately solved the problem to everyone’s satisfaction. How?

e. How can you cook an egg for exactly 15 minutes if all you have is a 7-minute hourglass and an 11-minute hourglass?

f. If you have 20 coins worth $1.35, and the coins are all nickels and dimes, how many do you have of each kind of coin?

g. Jocko ate a total of 100 peanuts over a period of 56 days. Each day he ate six more peanuts than on the previous day. How many peanuts did he eat on the first day?

h. You have six sections of a chain, each consisting of four links (see drawing). If the cost of cutting one link is 10 cents and welding it together again is 25 cents, what is the least it should cost to join the pieces. Tell how you will do the job.

i. If a cork and a bottle cost $2.10, and the bottle costs $2.00 more than the cork, what does the cork cost?

j. An employer has a thin seven-cm bar of gold. He paid an employee one cm. of gold every week for seven weeks. Pay day was on each Saturday. (1) If he makes only two cuts on the seven-cm bar, where did he cut it? (2) What are the lengths of the three pieces after the cut? (3) How would he use these three pieces to pay the employee one cm of gold each Saturday for the seven weeks?

k. What’s the smallest number of birds that could fly in this formation: 2 birds in front of a bird, 2 birds behind a bird, and a bird between 2 birds?

Many teachers find it useful to provide a “problem of the day” in a file folder in a math corner or a “problem of the week” on a bulletin board. Regardless of how you choose to emphasize the problem-solving process, the important goal is that we want to encourage children to do what is described on the following line.
Traveling from Fractions to Rational Expressions on the Number Line

Editor's Note: This is an article that is being published in two parts. Part two will follow in the next issue.

Historically, students have had a difficult time performing operations with rational expressions. This is especially problematic since rational expressions are incorporated in numerous mathematical topics. When students possess a deep understanding of numbers and the connection between numbers and variables, they should smoothly transition from one to the other. Since our goal is for students to better understand rational expressions, we need to be certain that our students understand fractions. Once the connection is made between the two, the students will become more confident using rational expressions. The following material is appropriate for Grade 9 in the Georgia Mathematics Curriculum standards. Specifically: MMIA2(e). Students will add, subtract, multiply, and divide rational expressions.

Applications to the Classroom:

Students who have difficulty performing operations with rational expressions need to be reminded of similar operations involving fractions. The following should serve as a reminder for your students.

<table>
<thead>
<tr>
<th>Renaming &amp; Simplifying Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concept Image for Fraction:</strong></td>
</tr>
<tr>
<td>![Fraction Image]</td>
</tr>
<tr>
<td>Ed is responsible for putting up signs on the side of a road two-thirds of a mile leading to the school carnival. The signs are to be spaced one-sixth mile apart from each other. How many signs will he put up?</td>
</tr>
<tr>
<td><strong>Rename</strong> ( \frac{2}{3} ) <strong>as a fraction having a denominator of 6.</strong></td>
</tr>
<tr>
<td>[ \frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6} ]</td>
</tr>
<tr>
<td><strong>Simplify.</strong> ( \frac{4}{6} )</td>
</tr>
<tr>
<td>[ \frac{4}{6} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{2}{3} ]</td>
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</tbody>
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<tr>
<th>Renaming &amp; Simplifying Rational Expressions</th>
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<tbody>
<tr>
<td><strong>Rename</strong> ( \frac{2}{3} ) <strong>as a fraction having a denominator of 3x.</strong></td>
</tr>
<tr>
<td>[ \frac{2}{3} = \frac{2 \cdot x}{3 \cdot x} = \frac{2x}{3x} ] Note: ( x \neq 0 )</td>
</tr>
<tr>
<td><strong>Simplify.</strong> ( \frac{2x}{3x} )</td>
</tr>
<tr>
<td>[ \frac{2x}{3x} = \frac{2 \cdot x}{3 \cdot x} = \frac{2}{3} ] Note: ( x \neq 0 )</td>
</tr>
</tbody>
</table>

| Rename \( \frac{x}{3} \) **as a fraction having a denominator of 6x(x+y).** |
| \[ \frac{x}{3} = \frac{x \cdot 2x(x+y)}{3 \cdot 2x(x+y)} = \frac{2x^2(x+y)}{6x(x+y)} \] Note: \( x \neq 0, x+y \neq 0 \) |
| **Simplify.** \( \frac{6x-9}{8x-12} \) |
| \[ \frac{6x-9}{8x-12} = \frac{3(2x-3)}{4(2x-3)} = \frac{3}{4} \] Note: \( x \neq \frac{3}{2} \) |
Adding Fractions with Like Denominators

Concept Image for Fraction:

Albert mowed one-fifth of the lawn while Hung-Hsi mowed two-fifths. Together, what part did they mow?

\[
\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5} = \frac{3}{5}
\]

Adding Rational Expressions with Like Denominators

\[
\frac{1}{x} + \frac{2}{x} = \frac{1+2}{x} = \frac{3}{x}
\]

Note: \(x \neq 0\)

Adding Fractions with Unlike Denominators

Concept Image for Fraction:

A local program has one-fourth hour of commercials and two-thirds hour of show time. How long is the entire show?

\[
\frac{1}{4} + \frac{2}{3} = \frac{1 \cdot 3 + 2 \cdot 4}{4 \cdot 3} = \frac{11}{12}
\]

Adding Rational Expressions with UnLike Denominators

\[
\frac{1}{x} + \frac{2}{xy} = \frac{1 \cdot y + 2 \cdot x}{x \cdot y} = \frac{y+2x}{xy}
\]

Note: \(x \neq 0, y \neq 0\)
NSF-Funded Explorations of Mathematics for High-School Teachers
Department of Mathematics, UGA
June 17–July 8, 2009

There will be NSF funding for approximately 7–10 teachers to have the opportunity to learn more about the relations between geometry and algebra. This should be of particular interest to teachers in the state of Georgia who are now teaching a curriculum that is based on an integrated approach to algebra and geometry.

**Title:** Explorations of Algebra and Geometry.

**Instructors:** Professor Ted Shifrin and Mo Hendon

**Dates:** June 17–July 8, 2009

We will explore the interplay of algebra (particularly linear algebra) and geometry, starting with the use of vector algebra to prove many classical theorems in Euclidean geometry—some well known, others less so. We will move on to study topics such as projective geometry and computer graphics, conic sections, and the question of how many lines intersect four general (mutually skew) lines in space. In part, participants will be encouraged to do some computer explorations.

Daily work will consist of three parts: a morning lecture, collaborative problem sessions, and later afternoon group work, thinking through the integration of algebra and geometry topics in the Math 1 and Math 2 curricula. Teachers will be asked to bring their texts and class materials from this school year and to work (with guidance) on strengthening and expanding these materials for use in the future.

**Funding:** A stipend of at least $600/week + housing assistance if needed (housing will be at Intown Suites Extended Stay, a fully furnished efficiency)

**Applications:** Each applicant should submit an application to Laura Ackerley by January 23, 2009. The application can be downloaded at the University of Georgia Mathematics Department website, [http://www.math.uga.edu](http://www.math.uga.edu). Please include in the application a list of math courses taught the last 5 years, a list of upper-division college mathematics courses taken, any extra information you wish us to have about you, and a 200-to-300-word “essay” on why this program is particularly of interest to you.

**Questions:** Please contact Ted Shifrin at shifrin@math.uga.edu
## Elementary Brain Teaser

### From Last Issue

**Circles and Squares**

Start with a square piece of paper. Draw the largest circle possible inside the square, cut it out and discard the trimmings. Draw the largest square possible inside the circle, cut the square out and discard the trimmings. What fraction of the original square piece of paper has been cut off and thrown away?

**Circles and Squares Solution:** 1/2 the area.

### New One!

**For Whom the Bell Tolls**

If a clock chimes 6 times in five seconds, how many times will it chime in ten seconds?

## Challenge Round

### From Last Issue

**How Big Is A Million?**

What is the sum of all the digits needed to write each counting number from 0 through 1,000,000? For example, the sum of all the digits needed to write each counting number from 18 through 23 would be: 

\[ 1+8+1+9+2+0+2+1+2+2+2+3 = 33. \]

**How Big Is A Million solution:** 27,000,001

You can pair the numbers as follows:

<table>
<thead>
<tr>
<th>Pair</th>
<th>Digit Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>999999</td>
</tr>
<tr>
<td>1</td>
<td>999998</td>
</tr>
<tr>
<td>2</td>
<td>999997</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>499999</td>
<td>500000</td>
</tr>
</tbody>
</table>

The sum of each of the digits of each of these 500,000 pairs is 54. Thus 54 x 500,000 = 27,000,000. But you must include 1,000,000. So the answer is 27,000,000 + 1.

### New One!

**Nine Digits**

By arranging the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 it is possible to come up with a fraction equivalent to one-eighth. For example:

\[ \frac{1}{8} = \frac{3187}{25496} \]

\[ 1/8 = 3187/25496 \] (can you write this as a true fraction in print?)

Your task is to arrange the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9 and come up with an equivalent fraction to one-fifth.
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Mail to: Susan Craig, GCTM Membership Director, 1011 Stewart Ave., Augusta, GA 30904-3151
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**Get Out Your Calendars, Day Planners, and PDAs**

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<th>Location</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>T³ Regional</td>
<td>Augusta, GA</td>
<td>February 6-7, 2009</td>
</tr>
<tr>
<td>NCTM</td>
<td>Annual Meeting</td>
<td>Washington, DC</td>
</tr>
<tr>
<td>NCTM Regional</td>
<td>Nashville, TN</td>
<td>November 18-20, 2009</td>
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**REFLECTIONS**

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